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ABSTRACT

This resource book contains 18 magic number tricks that spark the interest and imagination of students as they are led through a variety of mathematical computations and discoveries. Following each activity, students are asked to write about their discoveries and create their own magic tricks. A matrix of skills for all the activities and lists of the materials required for each are included. Each activity has four sections: (1) Performing the Trick, (2) Why the Trick Works, (3) Activity Pages, and (4) Using the Trick. The emphases in these activities are on student discovery, critical thinking, verbal and written communications, and creativity. The tricks work well in whole class or small group situations. (DDR)

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OPERATION MAGIC TRICKS

Ronald Edwards

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Introduction

Operation Magic Tricks contains a collection of magic number tricks that spark the interest and imagination of elementary students. As students are introduced to the number magic, they are led through a variety of mathematical computations and discoveries. After each activity, students are encouraged to write about their discoveries and to create their own collection of magic tricks.

For each activity, there is a list of skills necessary to perform the trick or to discover why the trick works. All skills are prealgebra level and are based on the four arithmetic operations or other elementary skills, such as consecutive counting or place value. The *MATRIX OF SKILLS* on the following page provides a summary of skills used throughout the book. This matrix is intended to assist teachers in selecting activities that introduce or reinforce specific mathematical concepts or skills.

A list of materials accompanies each activity. No specialized materials are needed; only common items like dice, coins, or markers are used. When tables, charts, or cards are required, reproducible blackline masters are provided with the activity.

The section *PERFORMING THE TRICK* presents detailed instructions that the teacher may use when presenting a number trick. Hints on presentation and methods of revealing predictions are included with each activity. The tricks require no special talents or sleight of hand in the presentations.

WHY THE TRICK WORKS provides a detailed explanation of the secret behind each trick. These discussions are written to be understood by elementary students. Teachers have the option of guiding students in discovering the math behind the magic or having students use the *ACTIVITY PAGES* provided. The activity pages help students unravel the mystery behind each trick by making observations and drawing conclusions. Students are encouraged to write their conclusions and to share them with classmates. Each activity ends with an exercise that either extends the given trick or suggests creating a new trick based on student discoveries.

The section *USING THE TRICK* offers suggestions for classroom presentation. Sample elementary mathematics exercises related to the specific concepts and skills listed for the trick are included. These exercises can be used to tie the magic tricks to lessons in the mathematics curriculum.

Although each trick is presented with a magician (teacher) working with a subject (student), all tricks can be performed within small groups or with an entire class. When students discover how a trick works, they should be encouraged to perform the trick with classmates, friends, or family members. In doing so, the students remember and follow a prescribed set of directions, practice mental arithmetic, and gain experience and confidence in communicating with others.

The emphasis in *Operation Magic Tricks* is on student discovery, critical thinking, communication (both verbal and written), and creativity. At each stage of this process, students should be given the opportunity to share and discuss their ideas. When students design and perfect a new trick, they should be given an opportunity to present their own magic routines.

to Gaeir Dietrich with great appreciation for her helpful insights and suggestions

Matrix of Mathematical Concepts and Skills

Concept or Skill	Activity																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
following directions	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
consecutive counting	√	√	√			√					√					√		
number names			√															
place value			√		√	√						√				√	√	√
expanded notation												√				√		
odd and even	√			√										√				
addition				√	√	√	√	√	√	√		√	√	√	√	√	√	√
subtraction						√							√			√	√	√
multiplication									√	√	√	√	√	√	√	√	√	
division													√	√				
multiples																		√
distributive property															√	√	√	
properties of nine						√												√
patterns		√	√		√	√	√	√	√	√	√	√			√			
using a matrix					√		√	√							√			
reading a flowchart																√		
base two												√						
sets											√		√					

NUMBER 1 HEADS OR TAILS

TEACHER'S INSTRUCTIONS

In this trick, your subject places a set of coins on a table. While your back is turned, your subject randomly flips over some of the coins and then hides one of the coins. You will predict whether the hidden coin is heads up or tails up.

SKILLS

- Consecutive counting
- Distinguishing odd and even

MATERIALS

- 10 to 20 coins

PERFORMING THE TRICK

1. **Ask your subject to take five or more coins and place them in a row on the table. It does not matter whether the coins are placed heads up or tails up.**
2. **Turn your back to the table, and ask your subject to turn over as many coins as he or she wishes. However, each time a coin is turned over, have your subject say "flip." Your subject can even flip the same coin more than once.**
3. **When your subject is finished flipping coins, have him or her cover one of the coins with a hand so that you cannot see it.**
4. **As you turn and face the table, you will predict whether the hidden coin is heads up or tails up. Of course, you have a 50% chance of being right, but if you follow the procedure below, your prediction will be correct 100% of the time.**

HOW THE TRICK WORKS

While your subject is placing the coins on the table, note how many of the coins are heads up. Try to be inconspicuous while counting.

When your back is turned, start counting consecutively from the number of heads you noted above. Count one each time your subject says "flip." After your subject finishes flipping coins, note whether your count ended with an odd or an even number. This fact will determine whether there is an odd or even number of heads showing on the coins on the table.

After your subject has covered one of the coins, turn to face the table. Since you already know whether there is an even or odd number of heads, inconspicuously note whether there is an odd or even number of heads showing on the coins. This will determine whether the hidden coin is heads up or tails up.

AN ILLUSTRATION

Suppose your subject has put nine coins on the table with heads and tails showing as pictured below.

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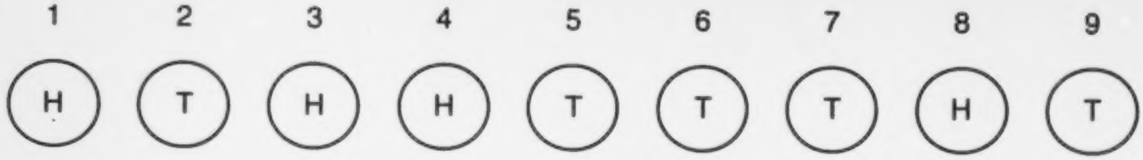
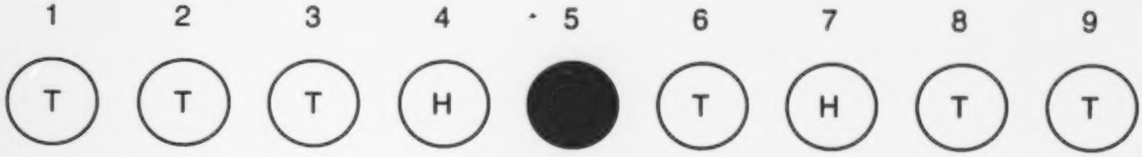


As you give directions to your subject, you note that there are 4 heads showing.

As your back is turned, suppose your subject flips coins 1, 3, 5, 7, 8, and then flips coin 5 a second time. Each time your subject says the word "flip," you count, starting from 4: 5, 6, 7, 8, 9, and 10. The last number (10) is even, which means that there should be an even number of heads showing on the coins at this time.

Meanwhile, your subject has covered one of these coins. Let us suppose that coin 5 is covered. As you face the table, you will see the following arrangement of coins:



From your count, you know that the number of heads should be even, and you see an even number (namely 2) on the table; therefore, the coin under your subject's hand must be tails up. When your subject's hand is removed from the coin, of course, your prediction is correct.

WHY THE TRICK ALWAYS WORKS

When you note the number of heads at the beginning of the trick, this number is either even or odd. Each time your subject flips a coin, you may not be sure whether the number of heads will be increased or decreased, but you are sure that the number of heads will change by one. Thus, for each flip, odd counts of heads will increase or decrease by 1, becoming even, and even counts will become odd. When your subject is finished flipping coins, you will know by your count whether there is an even or odd number of coins lying heads up.

When you turn, you will note whether there is an even or odd number of heads showing. Thus, you will know whether the hidden coin is heads up or tails up.

USING THE TRICK

This trick can be repeated several times with the same subjects if you vary the number of coins used and are careful about not giving away your prediction method. As a variation, the subject can toss each coin to determine if it will be heads up or tails up before placing it on the table.

Instead of coins you can use any type of counter that has two distinguishable faces. An exercise on activity page 5 suggests that students make counters to create their own version of the trick.

Activity pages 4–5 lead students through a discovery of the secret behind HEADS OR TAILS.

HEADS OR TAILS and the MATHEMATICS CURRICULUM

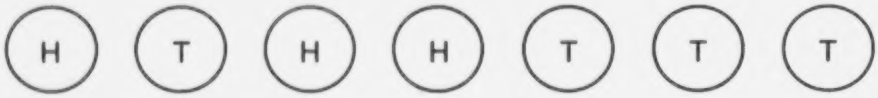
The magician's prediction in this trick is based on two mathematical skills: (1) consecutive counting from any given counting number and (2) distinguishing between even and odd numbers.

Below are some examples, taken from the elementary mathematics curriculum, of activities in counting and distinguishing even and odd. Note that students use counting when determining probabilities, as questions 5 and 6 illustrate.

1. In the sequences of counting numbers below, write the missing numbers in the blanks.
   - a. 1, 2, 3, ___, ___, ___
   - b. 3, 4, 5, ___, ___, ___
   - c. 12, 13, ___, ___, ___
2. Circle the even numbers in this list: 7, 4, 6, 9, 11, 14
3. Circle the odd numbers in this list: 1, 5, 8, 12, 3, 2, 9
4. Is there an even or odd number of black squares in the row pictured below?

   □ ■ □ ■ ■ □ ■ ■ □

5. Two coins are tossed. What is the probability that both coins land heads up?
6. Given a set of coins with heads up or tails up as shown below, one coin is selected at random. What is the probability that the coin is heads up?



ACTIVITY PAGES for HEADS OR TAILS

See if you can discover why the magician is 100% certain of his or her prediction.

HEADS OR TAILS TRICK

1. Ask your subject to take five or more coins and place them in a row on the table. It does not matter whether the coins are placed heads up or tails up.
2. Turn your back to the table and ask your subject to turn over as many coins as he or she wishes. However, each time a coin is turned over, have your subject say "flip." Your subject can even flip the same coin more than once.
3. When your subject is finished flipping coins, have him or her cover one of the coins with a hand so that you cannot see it.
4. As you turn and face the table, you will predict whether the hidden coin is heads up or tails up.

OBSERVATIONS

Place a set of coins in a row on the table with some coins heads up and some tails up.

1. Is the number of heads showing on the coins an even or an odd number?

2. Turn one of the coins over. Now, is the number of heads showing even or odd?

3. Turn over another coin. Is the number of heads showing even or odd?

4. Turn over several more coins. After each turn, keep track of whether the number of heads showing is even or odd. After the last turn, is the number of heads even or odd?

5. Cover one of the coins on the table. Count the number of heads on the uncovered coins. Is this number even or odd?

6. Do your answers in observations 4 and 5 give you a hint about the hidden coin?

CONCLUSION

1. Use your observations above to explain how the magician can make a correct prediction 100% of the time.

2. How does the magician know the number of heads at the beginning of the trick, the number of coins that are turned over, how many heads there are at the end of the trick, and what is hidden under the subject's hand?

MAKING YOUR OWN TRICK

Instead of coins, make your own disks. Mark the opposite sides of each disk in some way, perhaps using different colors.

Have your subject toss each disk and place the disks in a row as they fall to the table. After you turn your back to the table, see if you can devise a way to predict the color of a hidden disk without ever turning around to face the table.

NUMBER 2 THE DOMINO SHIFT

TEACHER'S INSTRUCTIONS

Faced with a row of dominoes which are facedown on a table, your subject moves several of them. Although your back is turned as the dominoes are moved, you will predict the number of dominoes moved by your subject.

SKILLS

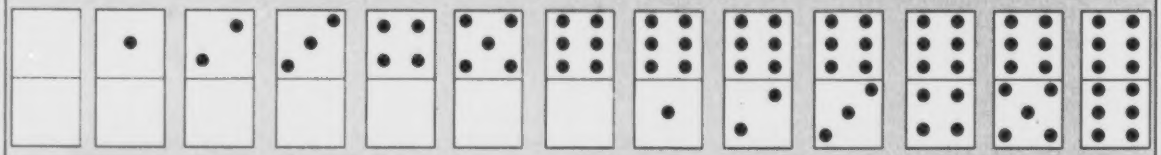
- Consecutive counting from a given number
- Constructing a model for a procedure

MATERIALS

- A double-six set of dominoes (a 28-piece set) or photocopy the domino template on page 12 and glue the "dominoes" to heavy cardboard

PERFORMING THE TRICK

From a double-six set of dominoes (28 pieces), place thirteen dominoes in a row as shown below. However, place them so the dots are facedown on the table. Note that the number of dots ranges from 0 to 12 as you move from left to right.



Choose a subject and explain that you want him or her to move some of the dominoes from the left end of the row to the right end, one domino at a time. Demonstrate this procedure by moving a few dominoes from left to right, one at a time. When you move them, keep the dominoes facedown on the table.

Now, turn your back to your subject and give directions 1–2 below.

1. Move one domino at a time from the left end of the line to the right end. Do this quietly so I will not know how many dominoes you are moving.
2. Let me know when you are finished moving the dominoes. Remember how many dominoes you have moved.
3. When your subject is finished, turn to face the table, and study the line of dominoes. Now, you can pick up one of the dominoes in the row, and the number of dots will match the number of dominoes that your subject has moved.

HOW THE TRICK WORKS

When you demonstrate how the dominoes should be moved, remember how many you move. This number will be your key for predicting the number moved by your subject.

After your subject has completed his or her moves, turn and count your key number from the right end of the row of dominoes. The domino you arrive at will predict the number of moves made by your subject.

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AN ILLUSTRATION

In the illustration below, the numbers (0 to 12) indicate the number of dots on the dominoes. Of course, you and your subject will only see the blank backs of the dominoes. When you set up the trick, the dominoes appear in the order shown below.

0 1 2 3 4 5 6 7 8 9 10 11 12

As you explain the trick to your subject, suppose you move two (this will be your key number) dominoes from the left end to the right end of the row. After this, the dominoes will be in the order shown below.

2 3 4 5 6 7 8 9 10 11 12 0 1

When you turn your back to the table, suppose your subject moves five dominoes from left to the right. Now, the sequence of dots will appear as shown below.

7 8 9 10 11 12 0 1 2 3 4 5 6

When you turn and look at the dominoes, appear to be studying them for a clue. In a reasonable amount of time, pick up the second (your key number) domino from the right end of the row. This domino has five dots. Ask your subject, "Is this the number of dominoes you moved?" Of course, it is.

You can vary your key number each time you repeat the trick. In this way, you are not always picking up the same domino.

WHY THE TRICK WORKS

Pick any number as your key, let us say 4. After you transfer four dominoes, the sequence will appear as follows: 4 5 6 7 8 9 10 11 12 0 1 2 3.

Suppose your subject moves any number of dominoes up to nine. Note that the subject may try to fool the magician by moving zero dominoes. In the table below, each possible move by the subject is examined.

DOMINOES MOVED BY THE SUBJECT	NUMERICAL ORDER OF THE DOMINOES													YOUR PREDICTION (ROW MARKED *) (KEY # = 4)
0	4	5	6	7	8	9	10	11	12	0*	1	2	3	0
1	5	6	7	8	9	10	11	12	0	1*	2	3	4	1
2	6	7	8	9	10	11	12	0	1	2*	3	4	5	2
3	7	8	9	10	11	12	0	1	2	3*	4	5	6	3
4	8	9	10	11	12	0	1	2	3	4*	5	6	7	4
5	9	10	11	12	0	1	2	3	4	5*	6	7	8	5
6	10	11	12	0	1	2	3	4	5	6*	7	8	9	6
7	11	12	0	1	2	3	4	5	6	7*	8	9	10	7
8	12	0	1	2	3	4	5	6	7	8*	9	10	11	8
9	0	1	2	3	4	5	6	7	8	9*	10	11	12	9

No matter what key number you use, the prediction row in the table will indicate how many dominoes your subject moved.

There is one difficulty that you might encounter when performing the trick. The number of dots on the dominoes only goes up to twelve. Therefore, if your subject decides to move more than twelve dominoes, your key number will not match the number moved. Try moving more than twelve dominoes and see what happens to your prediction. Of course, you can prevent this from happening by putting limits on the number of dominoes your subject can move.

USING THE TRICK

When you repeat THE DOMINO SHIFT, vary the number of dominoes you move so that your key number will not always be the same. Also, note that you must move at least one domino in your demonstration in order to get the "zero" to the right end of the line. If you have a larger set of dominoes (double-nine set) you can place more dominoes in your row; of course, you can also decrease the number of dominoes used.

If you disguise your prediction, it will be difficult for your subject to discover how the trick works. The activity pages (pp. 10–11) lead students to a discovery of this secret. An exercise on activity page 11 makes suggestions for students to alter the presentation of the trick.

In the models we have used to demonstrate this trick so far, we have focused on using a key number to tell how many dominoes the subject has moved. However, the number on the domino being moved can be used to tell the total number of dominoes moved (by both you and your subject). If you add one to the value of the domino being moved, the result will tell you the total number of dominoes moved up to that point. For example, if your subject is moving domino 7, then that is the 8th (7 + 1) domino to be moved. You and your students can explore ways to create a new trick based on this information.

THE DOMINO SHIFT and the MATHEMATICS CURRICULUM

The prediction key to this trick (picking up the correct domino) is based on consecutive counting and constructing visual models for the domino transfers. Curriculum examples for consecutive counting are included in HEADS OR TAILS, see page 3.

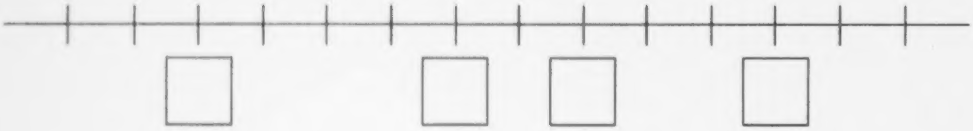
Below are some curriculum illustrations of visual counting models using number lines and further examples of counting techniques.

1. Write the missing numbers in the boxes. In the circle, tell how many numbers there are in the sequence 3, 4, 5, ... 9.

3 4 5 ☐ ☐ ☐ 9 ○

2. The four given numbers below belong on the number line. Write each number in the correct box.

12 7 9 3



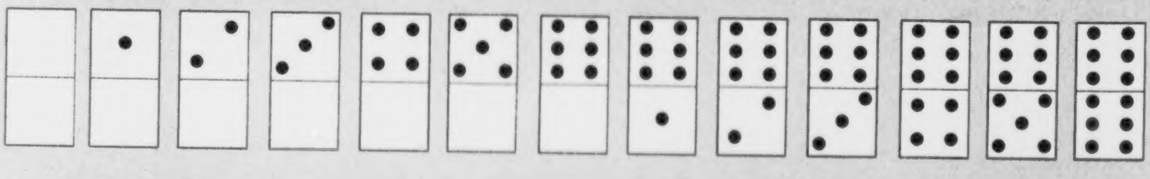
3. How many numbers are there from 0 to 7 (including 0 and 7)? From 0 to 2? From 3 to 7?

ACTIVITY PAGES for THE DOMINO SHIFT

See if you can determine how the magician always picks up the correct domino.

THE DOMINO SHIFT TRICK

From a double-six set of dominoes (28 pieces), place thirteen dominoes in a row as shown below. However, place them so the dots are facedown on the table. Note that the number of dots ranges from 0 to 12 as you move from left to right.



Choose a subject and explain that you want him or her to move some of the dominoes from the left end of the row to the right end, one domino at a time. Demonstrate this procedure by moving a few dominoes from left to right, one at a time. When you move them, keep the dominoes facedown on the table.

Now, turn your back to your subject and give directions 1–2 below.

1. Move one domino at a time from the left end of the line to the right end. Do this quietly, so I will not know how many dominoes you are moving.
2. Let me know when you are finished moving the dominoes. Remember how many dominoes you have moved.
3. When your subject is finished, turn to face the table, and study the line of dominoes. Now, you can pick up one of the dominoes in the row, and the number of dots will match the number of dominoes that your subject has moved.

OBSERVATIONS

Suppose that when the trick is set up, the dominoes appear in the order shown below, but facedown on the table so that you cannot see the numbers.

0 1 2 3 4 5 6 7 8 9 10 11 12

1. Choose a small number to indicate the number of dominoes moved by the magician when demonstrating the trick. Call this number your "key number." After the key number of dominoes have been moved by the magician, list the new numerical sequence of the 13 dominoes.

2. Suppose the subject decides to move any number of dominoes from zero to nine. Complete the table below for each of these possible moves by the subject.

DOMINOES MOVED BY THE SUBJECT	NUMERICAL ORDER OF THE DOMINOES AFTER THE SUBJECT'S MOVE	COUNT THE KEY NUMBER FROM THE RIGHT END AND NAME THE DOMINO
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

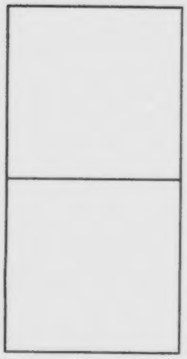
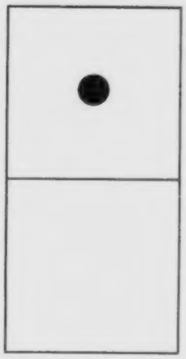
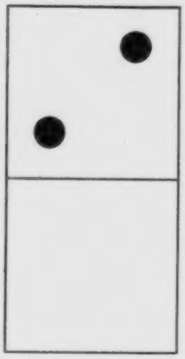
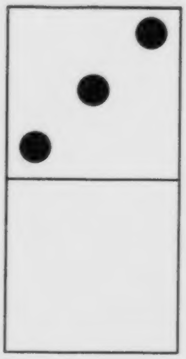
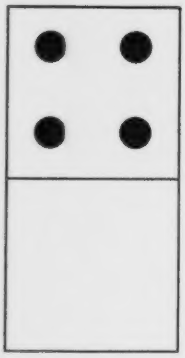
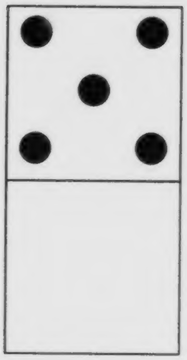
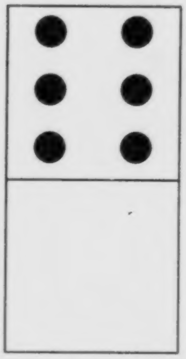
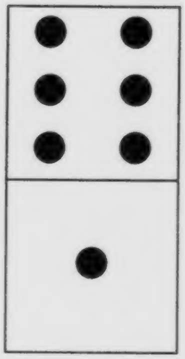
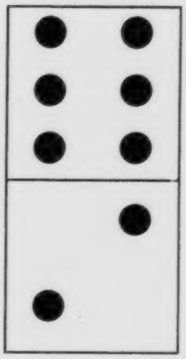
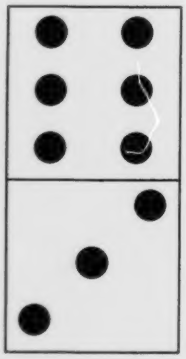
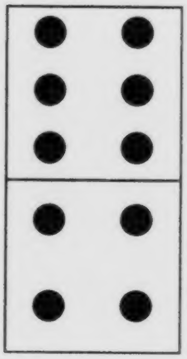
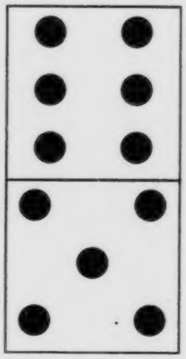
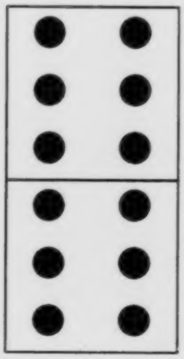
3. What do you notice about the numbers in the first and last rows of the table above?

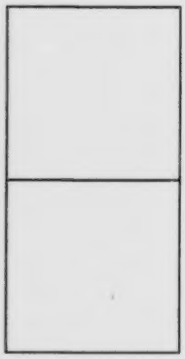
CONCLUSION

1. Explain how the magician knows which domino to pick up in order to predict how many dominoes were moved by the subject. What would happen if the subject moved a large number of dominoes (over twelve)?

MAKING YOUR OWN TRICK

Make necessary changes in the trick so that a deck of cards could be used in place of the dominoes. How many cards could you use? Explain how the original row of cards should be placed on the table and how the magician will make the correct prediction.



19 EXTRAS

NUMBER 3 THE GREAT TRICK

TEACHER'S INSTRUCTIONS

In this trick, you will miraculously discover a number that your subject selects from a set of five cards.

SKILLS

- Consecutive counting
- Reading two-digit numbers
- Spelling number names
- Discovering patterns

MATERIALS

- The set of five cards (a reproducible blackline master for the cards is on page 19)

PERFORMING THE TRICK

Before performing this trick, you must prepare the set of five cards. On one side of each card is a letter, and on the reverse side is a two-digit number. Make sure that you cannot see through the card. That would give away the trick.

To prepare your cards, copy the master on page 19, cut out the letters and numbers, and paste them onto opposite sides of cardboard squares. Make sure that the correct numbers and letters are matched on each card. With your set of prepared cards, follow the steps below.

1. Randomly place the five cards in a row with the two-digit numbers face up.
2. Ask your subject secretly to choose and remember one of the two-digit numbers.
3. Now, ask your subject to shuffle the cards and to place them randomly in a row with the letters face up.
4. As you touch various letters on the cards, ask your subject to spell out his or her number silently, one letter for each card that you touch. It is important that this spelling be correct, so you might want to make a reference list available.

   13—THIRTEEN

   16—SIXTEEN

   27—TWENTY-SEVEN

   45—FORTY-FIVE

   72—SEVENTY-TWO

5. Tell your subject to say "STOP" when the spelling is completed. When your subject says "STOP," turn over the card that you are touching. The number on the reverse side of this card will be the number which was selected by your subject. You will always be pointing to the correct card if you followed the steps above.

BEST COPY AVAILABLE

HOW THE TRICK WORKS

After your subject places the cards in a row with the letters faceup, start by touching any six of the cards (e.g., A, R, E, T, A, G). Let your finger rest for a moment on each card to allow your subject to spell out his or her number silently. From this point on, the key word is GREAT. For the seventh, eighth, ninth, tenth, and eleventh cards that you touch, follow the order G, R, E, A, and T.

Your subject will stop you at one of these letters. On the reverse side of this card will be your subject's number.

AN ILLUSTRATION

Suppose your subject has selected the number 72 (SEVENTY-TWO) and after shuffling the cards has arranged them on the table as shown below.

R E G T A

First, you touch any six cards, for example, E, T, G, R, T, and A. As you touch these six cards, your subject silently spells S, E, V, E, N, and T. After the sixth card, you must point to the cards in the following order: G, R, E, A, and T. As you point to G, R, E, and A, your subject will silently spell Y, T, W, and O. At this point, your subject will say "STOP." This will be the tenth card since there are ten letters in SEVENTY-TWO. When stopped, you will be touching the A. When you turn this card over, the subject's number, 72, will appear.

WHY THE TRICK ALWAYS WORKS

In order to perform this trick, it is not necessary to understand the "magic" behind the trick. But, most students will want to know, Why does this trick always work?

The key word is important, and you must remember it. In this example, the key word is GREAT. In the table below are the letters in the key word, the two-digit numbers written on the reverse of these letters, the spellings for each two-digit number, and the number of letters in each spelling (ignore the hyphens).

LETTER	2 DIGITS	SPELLING	# OF LETTERS IN SPELLING
G	16	sixteen	7
R	13	thirteen	8
E	45	forty-five	9
A	72	seventy-two	10
T	27	twenty-seven	11

Note that the number selected by your subject will have 7, 8, 9, 10, or 11 letters in its spelling. As you point to the first six cards, your subject goes through the first six letters in the spelling of his or her number.

When you next point to G, your subject will stop you if the number selected was 16, which has 7 letters. Of course, the card you are touching, G, will match the 16. The same is true when you point to R. If the subject's number was 13, you will be stopped at the R since THIRTEEN has 8 letters. The same is true for 45, 72, and 27.

You can change the key word and even add more cards to your trick. For example, you could add a sixth card with an S on one side and 78, which has 12 letters in its spelling, on the reverse side. Now, your key word would be "GREATS."

USING THE TRICK

If you perform the trick with a small group of students, you can make your cards with small squares of cardboard or blocks of wood. Large cards, which can be seen from the back of the classroom, should be used if the trick is performed before an entire class.

After performing THE GREAT TRICK successfully several times, students will want to know how the trick works. You can explore this question with the class by using the activity pages (pp. 16–18) to lead students through a discovery of the trick's secret. Once students understand the magic behind the trick, individual students or groups can devise their own trick based on the same counting principles. A suggestion for such a trick appears on activity page 18.

THE GREAT TRICK and the MATHEMATICS CURRICULUM

The secret behind this trick is based on three mathematical skills: counting, reading numbers, and writing number names. Below are some sample activities that address these skills.

1. Find the number of letters in the box. Write the answer in the circle.

A	i	$	?	5
m	t	s	=	4
c	u	E	r	9
M	S	a	p	H

2. Circle the numeral below that is represented by the following words: two million, eighty thousand, seven hundred five.

   a. 2,800,705

   b. 280,705

   c. 2,080,705

3. Write the numeral 107 in words.

ACTIVITY PAGES for THE GREAT TRICK

See if you can discover why this trick will always work.

THE GREAT TRICK

1. Randomly place the five cards in a row with the two-digit numbers face up.
2. Ask your subject secretly to choose and remember one of the two-digit numbers.
3. Now, ask your subject to shuffle the cards and to place them randomly in a row with the letters face up.
4. As you touch various letters on the cards, ask your subject to spell out his or her number silently, one letter for each card that you touch. It is important that this spelling be correct, so you might want to make a reference list available.

   13—THIRTEEN

   16—SIXTEEN

   27—TWENTY-SEVEN

   45—FORTY-FIVE

   72—SEVENTY-TWO
5. Tell your subject to say "STOP" when the spelling is completed. When your subject says "STOP," turn over the card that you are touching. The number on the reverse side of this card will be the number which was selected by your subject.

OBSERVATIONS

1. The five letters used in the trick spell the word GREAT. Fill in the blanks in the table below.

LETTERS USED ON THE CARDS	NUMBER ON CARD'S BACK	SPELLING OF THE NUMBER NAME	NUMBER OF LETTERS IN THE SPELLING
G	16	sixteen	7
R	________	________________	________
E	________	________________	________
A	________	________________	________
T	________	________________	________

BEST COPY AVAILABLE

2. Describe the number pattern in the last column in the table.

3. How many cards will the magician touch if you choose 16 as your number?

4. What is the last letter touched by the magician if you choose 16 as your number?

5. Answer questions 3 and 4 above if your choice was 13.

6. Answer questions 3 and 4 above if your choice was 45 or 72 or 27.

7. Does it matter which cards the magician touches the first six times?

8. List the letter order in which the magician should touch the seventh, eighth, ninth, tenth, and eleventh cards so that he or she will end on the subject's number.

CONCLUSION

1. Write a brief explanation of why this trick works.

2. Why must the magician remember the word GREAT?

3. In what order should the magician point to the letters on the cards?

4. What is the largest number of letters that the magician will have to touch?

MAKING YOUR OWN TRICK

Make a list of objects in a room, but make sure that the number of letters used in spelling the names of these objects follow in sequence, such as 3 letters, 4 letters, 5 letters, 6 letters, and so forth. Such a list might start as follows: PEN (3 letters), BOOK (4 letters), CHAIR (5 letters), WINDOW (6 letters). Make the list as long as you like.

Devise a trick in which you name a list of objects in the room and ask your subject to remember one of these objects. As you point to different objects in the room, have your subject spell out the name of the object he or she has chosen—one letter for each object. When the spelling is completed, have your subject say "STOP." You should be pointing to the object selected by your subject.

To make the trick effective, make a varied list of objects and make the order in which you point to objects in the room seem random. But, as your "key," you must remember the sequence used in making your list—for example, PEN, BOOK, and CHAIR in the illustration above.

Try out your new trick with a friend until it works smoothly. Now, try performing it in class or at home with friends and family members.

CARD 1	CARD 2	CARD 3	CARD 4	CARD 5
G	R	E	A	T
16	13	45	72	27

NUMBER 4 SUM OF YOUR CARDS

TEACHER'S INSTRUCTIONS

In this amazing trick, your subject selects six of twelve numbers which appear on cards and places the numbers faceup on the table. During the entire trick your back is turned to your subject, yet you will compute the sum of the six unseen numbers.

SKILLS

- Add a column of six one- or two-digit numbers
- Distinguish between even and odd numbers

MATERIALS

- A set of six prepared cards (a reproducible blackline master is on page 25)

PERFORMING THE TRICK

Give the six prepared cards to your subject, and then turn your back to your subject for the remainder of the trick as you give your subject the directions below.

THE SIX CARDS

	Card 1	Card 2	Card 3	Card 4	Card 5	Card 6
Front	1	3	5	7	9	11
Back	2	4	6	8	10	12

1. Have your subject select six of the twelve numbers which appear on the cards. Place the cards on the table so that the chosen numbers are faceup.
2. Ask your subject, How many of the chosen numbers are even numbers?
3. Have the subject add the six chosen numbers and write this sum on a piece of paper. (If a calculator is used, your subject can leave the sum on the calculator display.)
4. At this point, you can announce the sum of the six numbers that you have not seen. Your prediction will match your subject's sum if you follow the simple computation below.

HOW THE TRICK WORKS

When your subject tells you how many of the chosen numbers are even, add this number to 36 and that is your prediction.

BEST COPY AVAILABLE

AN ILLUSTRATION

Suppose your subject has made the selection pictured below.

1 3 6 7 9 12

Your subject tells you that two of the numbers are even. As your subject adds the six numbers that are faceup, you mentally compute your prediction: 2 + 36 = 38. When your subject announces that the sum is complete, you announce that the sum is 38 and, of course, you are right.

WHY THE TRICK ALWAYS WORKS

If you place the six cards on the table so that the odd numbers are faceup, the sum of these numbers is 1 + 3 + 5 + 7 + 9 + 11 or 36. Now, suppose you turn over one of the cards to expose the even number on the card's back. Note that the sum, 36, will be increased by 1. If you turn over another odd number, again the sum (now 37) will be increased by 1. This pattern will continue for each odd-numbered card that you choose to turn over. Therefore, if you add the number of even-numbered cards that are showing to 36, you will have the sum of the six cards that are left faceup on the table.

USING THE TRICK

When working with individuals or small groups, you can duplicate the blackline master on page 25 and paste the numbers on opposite sides of card stock. If you perform the trick before a class, a larger set of cards should be prepared.

It is possible that after this trick is performed several times, some students may guess how you make your prediction. The activity pages (pp. 23–24) lead students through a discovery of the trick's secret. Also, an exercise on activity page 24 challenges students to expand the trick by increasing the number of cards used.

SUM OF YOUR CARDS and the MATHEMATICS CURRICULUM

The "magic" behind this trick is related to summing consecutive counting numbers, summing consecutive odd counting numbers, and summing consecutive even counting numbers. Below are some sample exercises from the elementary mathematics curriculum that explore the relationships of these computations.

1. Add the consecutive counting numbers starting with 1 and going up to the following numbers:

   a. 3

   b. 4

   c. 5

   d. 6

   e. 7

   f. 8

2. Use the pattern of your answers in 1 above to predict the following:
   a. the sum of the first nine counting numbers

   b. the sum of the first twelve counting numbers

3. Find the following sums of consecutive odd counting numbers:
   a. $1 + 3 =$

   b. $1 + 3 + 5 =$

   c. $1 + 3 + 5 + 7 =$

   d. $1 + 3 + 5 + 7 + 9 =$

   e. $1 + 3 + 5 + 7 + 9 + 11 =$

4. Use your answers in 3 above to state a general rule for adding consecutive odd counting numbers.

5. How could you use the answers in 3 above to determine sums of consecutive even counting numbers?

ACTIVITY PAGES for SUM OF YOUR CARDS

The magician never sees the six numbers selected yet is able to predict the sum. See if you can discover the magician's secret.

SUM OF YOUR CARDS TRICK

Give the six prepared cards to your subject, and then turn your back to your subject for the remainder of the trick as you give your subject the directions below.

THE SIX CARDS

	Card 1	Card 2	Card 3	Card 4	Card 5	Card 6
Front	1	3	5	7	9	11
Back	2	4	6	8	10	12

1. **Have your subject select six of the twelve numbers which appear on the cards. Place the cards on the table so that the chosen numbers are faceup.**
2. **Ask your subject, How many of the chosen numbers are even numbers?**
3. **Have the subject add the six chosen numbers and write this sum on a piece of paper. (If a calculator is used, your subject can leave the sum on the calculator display.)**
4. **At this point, you can announce the sum of the six numbers that you have not seen.**

OBSERVATIONS

1. Place the six cards so that all the odd numbers are faceup. What is the sum of these six numbers?

2. Now, turn over one of the cards. How many even numbers are now faceup? What is the sum of the six numbers now showing?

3. Return the cards so only odd numbers are showing. Turn over two of the cards. How many even numbers are now faceup? What is the sum of the six numbers now showing?

4. Again, repeat the process above, but this time turn over three cards. What is the sum?

CONCLUSION

1. Suppose you are the magician and your subject has selected six numbers and has just told you how many of the numbers are even. Explain how you could predict the sum of the six numbers that are faceup.

MAKING YOUR OWN TRICK

Add one or more cards to the six used in the trick. Write the rule for making your prediction of the sum for the new set of cards.

How many new cards have you added?

What numbers should you put on each of the new cards?

What is the sum of all the odd numbers on your new set of cards?

	Card 1	Card 2	Card 3	Card 4	Card 5	Card 6
Front	1	3	5	7	9	11
Back	2	4	6	8	10	12

NUMBER 5 THE MEMORY TRICK

TEACHER'S INSTRUCTIONS

The aim of this trick is to impress others with your highly developed memory, or at least to make people believe that you have such memory powers.

SKILLS

- Use row and column numbers to locate an entry in a table
- Memorize a list of four simple rules
- Mentally compute the sum of two single-digit numbers
- Mentally add 9 to a two-digit number

MATERIALS

- Before performing the trick, you will need to reproduce the table on page 32. When you perform the trick with an individual, the table can be affixed to a card. If you perform the trick before a class, a transparency can be prepared for the overhead projector.

PERFORMING THE TRICK

1. Hand the prepared card to your subject, or project the table using an overhead projector.
2. Announce that you have memorized all thirty-six, five-digit numbers on the card. Each box on the card can be identified by stating its row number and its column number; for example, row 3 and column 4 identifies the box with the number 34718.
3. Before proceeding, make sure your subject (or class) understands how to use row and column numbers to locate entries in the table.
4. Ask your subject to give you the row and column numbers of any box, and you will recall the five-digit number located in that box. Of course, during this entire trick you will not look at the table. If the table is projected, turn your back to the screen.
5. If a blackboard is available, you can write the five-digit number on the board. Otherwise, you can write it on a pad of paper or simply announce the five digits. The trick can appear more dramatic if you close your eyes and appear to be searching your mind for the number. Of course, you need not memorize any of the numbers on the card, for there is a trick to producing the number in each box. All you need to do is remember the rules listed below.

HOW THE TRICK WORKS

When your subject gives you the row and column numbers, form a two-digit number with these digits (row first, then column) and carry out the following mental computations:

1. Add 9 to the two-digit number. Then reverse the digits in the sum, unless the units digit is a zero. These will be the first two digits in the box.

BEST COPY AVAILABLE

For example, for row 4 and column 5, you compute 45 + 9 = 54 and reverse the digits, giving 45 as the first two digits.

2. Sum the first two digits to find the third digit. However, if the sum is ten or more just write the units digit of the sum.

   In the example, 4 + 5 = 9, which is the third digit in the box.

3. Sum the second and third digits to get the fourth digit.

   In the example, 5 + 9 = 14, so dropping the 1 from the tens place, we have 4 as the fourth digit.

4. Add the third and fourth digits to get the fifth digit.

   In the example, 9 + 4 = 13, so dropping the 1 from the tens place, we have 3 as the fifth digit. Thus, the number in row 4 and column 5 is **45943**.

With a little practice, you will be able to write or announce the five-digit number quickly and accurately.

AN ILLUSTRATION

Suppose your subject wishes you to recall the five-digit number in row 2 and column 1.

You start with 21 (the row and column numbers) and add 9, which gives you 30. Since there is a 0 in the units place of this number, do not reverse the digits. Thus, the first two digits are **30**.

Add these digits: 3 + 0 = 3, which is the third digit. Thus, the first three digits are **303**.

Add the last two digits: 0 + 3 = 3, which is the fourth digit. Thus, the first four digits are **3033**.

Finally, add 3 + 3 = 6 for the last digit. Thus, the five-digit number is **30336**.

WHY THE TRICK ALWAYS WORKS

The reason this trick always works is very simple. In designing this trick, we start with a blank table with 6 rows and 6 columns. Next, a set of four rules were established in order to produce the numbers in the 36 boxes in the table. These rules were designed so they would be easy to remember and so that the computations would be simple enough to be done mentally. Finally, the numbers in the table were determined using the four rules and were written in the proper row and column locations. The magician does not have to memorize the numbers but must remember the rules for producing the numbers.

USING THIS TRICK

The table on page 32 can be duplicated and glued to a piece of cardboard if you are performing the trick with individual subjects. If you plan to perform the trick in front of a class, a transparency of the table can be used. In this way, all students can follow the trick and see if your memory is as good as you claim it is.

The first few times you perform this trick in a class, the students should be impressed. We do not exercise our memory as we used to in schools.

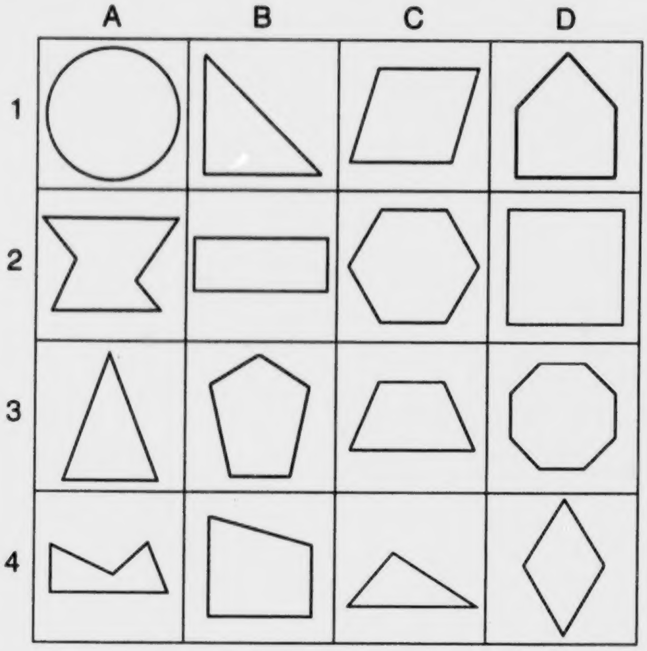
When students discover how the trick works (see activity pages 29–31) or you share this information with them, they will be able to perform the trick with friends or family members. In performing the trick, students will be doing mental arithmetic, remembering a set of rules to be followed, and gaining confidence in communicating ideas with others.

In an exercise on activity page 31, students are encouraged to create their own memory trick.

THE MEMORY TRICK and the MATHEMATICS CURRICULUM

Performance and understanding of this trick is based on working with arrays, doing mental arithmetic, and forming sequences. Below are some mathematical activities addressing these skills.

1. In the chart to the right, rows are named with numbers, and columns with letters.
   a. Draw the figure that is in location B3.
   b. In what location do you find the circle?



2. Use the rules below with the following set of numbers: 34, 58, and 27.

   Rule 1: Add nine to the given number.

   Rule 2: Add ten to the given number and then subtract one from the result.

3. Examine the following sequence of counting numbers:

   1, 1, 2, 3, 5, 8, 13, __, __, __, ...

   a. What numbers belong in the three blanks?

   b. Verbally describe how the sequence is formed.

ACTIVITY PAGES for THE MEMORY TRICK

In this trick, the magician does not memorize the numbers in the table. See if you can discover how the magician finds the numbers in each box.

THE MEMORY TRICK

1. Hand the prepared card to your subject.
2. Announce that you have memorized all thirty-six, five-digit numbers on the card. Each box on the card can be identified by stating its row number and its column number; for example, row 3 and column 4 identifies the box with the number 34718.
3. Before proceeding, make sure your subject understands how to use row and column numbers to locate entries in the table.
4. Ask your subject to give you the row and column numbers of any box, and you will recall the five-digit number located in that box. Of course, during this entire trick you will not look at the table.
5. If a blackboard is available, you can write the five-digit number on the board. Otherwise, you can write it on a pad of paper or simply announce the five digits. The trick can appear more dramatic if you close your eyes and appear to be searching your mind for the number.

OBSERVATIONS

1. Look at the second column of numbers in the table. How can you find the third number in each box by using the first two numbers?
2. Does this method work for all numbers in the table? Look at the number in row 6, column 4. How could 3 and 7 give you a 0 as the third number?
3. How could the second and third numbers in each box be used to find the fourth number?
4. How could the third and fourth numbers be used to find the fifth number?

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5. Take any number in columns 2 through 6 and reverse the first two digits. Now, subtract 9. What do you notice about your answer and the location of the number (row and column numbers)?

6. Based on your observation in 5 above, how could you find the first two numbers in each box by using the row and column numbers?

7. Look at the numbers in column 1 in the table. Subtract 9 from the first two digits of each of these numbers. What do you notice about the answers?

8. Why do you think that the digits were not reversed for the numbers in column 1?

CONCLUSION

1. Write a set of rules that could be used to predict the number in each box in the table if you know the row number and column number.

2. What is the rule for the first two digits? How about numbers in column 1?

3. What are the rules for the third, fourth, and fifth digits?

MAKING YOUR OWN TRICK

When you know how THE MEMORY TRICK works, you can produce your own variation on the trick.

The figure below contains thirty-six empty boxes. You are to devise a method of producing six-digit numbers for each of these boxes. You may use rules similar to those used in THE MEMORY TRICK. However, it would be more interesting if you made up your own rules, for then only you will know how the trick works.

Now, use your rules to fill in the boxes in the figure below, and try performing your trick on other people.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

COLUMN NUMBERS

ROW NUMBERS

	1	2	3	4	5	6
1	20224	12358	22460	32572	42684	52796
2	30336	13471	23583	33695	43707	53819
3	40448	14594	24606	34718	44820	54932
4	50550	15617	25729	35831	45943	55055
5	60662	16730	26842	36954	46066	56178
6	70774	17853	27965	37077	47189	57291

NUMBER 6 MAGIC NINE

TEACHER'S INSTRUCTIONS

There are numerous magic tricks that make use of special properties of the number nine. Here is such a trick, but in this one, the number nine is well hidden.

SKILLS

- Place value (tens and ones)
- Addition facts (nine)
- Subtraction of single-digit numbers

MATERIALS

- 20 pennies

PERFORMING THE TRICK

Place twenty pennies on a table in front of your subject. Now, turn your back to the table while you give directions 1–4 below to your subject.

1. **Pick up a few pennies (any single-digit number) and put them in your pocket.**
2. **Count the number of pennies left on the table. Sum the digits in this number.**
3. **Pick up the number of pennies corresponding to this sum. Put them in your pocket, also.**
4. **Pick up some more pennies, but hold these pennies in a closed fist in front of you. Tell me when you are done.**
5. **When your subject is ready, turn and face him or her while you announce how many pennies are concealed in his or her fist and add, "You can keep the eleven cents in your pocket." This announcement is quite effective if you say it immediately upon turning to your subject.**

HOW THE TRICK WORKS

There will always be eleven cents in your subject's pocket. To determine the number in your subject's fist, as you slowly turn, glance at the table and note the number of pennies. As soon as you do this, look at your subject so it is not obvious that you counted the pennies on the table.

Mentally subtract this number from nine and that will be the number of pennies in your subject's fist. Thus, you can announce the number held in the closed fist and tell your subject to keep the eleven cents that was pocketed.

AN ILLUSTRATION

As your back is turned, suppose your subject picks up six pennies and places them in his or her pocket. Fourteen pennies remain on the table.

Your subject adds the digits in 14 (1 + 4 = 5) and removes five more pennies from the table, placing them in his or her pocket. This leaves nine pennies on the table. This is the "magic nine."

BEST COPY AVAILABLE

Now, your subject picks up some more pennies. Suppose your subject picks up three pennies and holds them in his or her fist, as directed. This leaves six pennies remaining on the table when you turn to face your subject.

As you turn, you note the six pennies on the table; subtract 6 from 9 and announce, "You are holding three pennies in your hand. I'll take those, but you can keep the eleven cents in your pocket." You only say this if you do not mind giving away eleven cents.

WHY THE TRICK WORKS

No matter how many pennies are taken by your subject in step 1 of the trick, there will be nine pennies left on the table at the end of step 3. To prove that this is true, the table below lists every possible selection that the subject can make, from one to nine pennies.

NUMBER TAKEN IN STEP 1	NUMBER LEFT IN STEP 2	NUMBER REMOVED IN STEP 3	NUMBER ON TABLE AFTER STEP 3
1	20 – 1 = 19	1 + 9 = 10	19 – 10 = 9
2	20 – 2 = 18	1 + 8 = 9	18 – 9 = 9
3	20 – 3 = 17	1 + 7 = 8	17 – 8 = 9
4	20 – 4 = 16	1 + 6 = 7	16 – 7 = 9
5	20 – 5 = 15	1 + 5 = 6	15 – 6 = 9
6	20 – 6 = 14	1 + 4 = 5	14 – 5 = 9
7	20 – 7 = 13	1 + 3 = 4	13 – 4 = 9
8	20 – 8 = 12	1 + 2 = 3	12 – 3 = 9
9	20 – 9 = 11	1 + 1 = 2	11 – 2 = 9

The table above illustrates one of the special properties of the number nine. Note that nine pennies are left on the table at the end of step 3 of the trick. Of course, this means that there are eleven pennies in the subject's pocket.

Now, in step 4 of the trick, the subject takes some of the remaining nine pennies to conceal in his or her fist. As soon as you turn and see the number left on the table, nine minus this number will be the number of pennies in the subject's fist.

USING THE TRICK

The MAGIC NINE trick can be done using any set of twenty objects. You may not want your students to use a book of matches, but this works nicely.

As long as you disguise glancing at the remaining pennies on the table, your subject will have a hard time figuring out how you made your prediction. Of course, if you repeat the trick and call attention to the fact that there are eleven pennies in your subject's pocket, this will provide a clue to the secret.

The activity pages (pp. 36–37) lead the student to the discovery of the special property that leaves nine pennies on the table. When this fact is known, the student will know the prediction rule.

Could more than 20 pennies be used in the trick? An exercise on activity page 37 explores this possibility.

THE MAGIC NINE TRICK and the MATHEMATICS CURRICULUM

Some of the special properties of nine are illustrated below in exercises from the mathematics curriculum.

1. A number is evenly divisible by 9 if the sum of the digits of the number is divisible by 9. Which of the following numbers are divisible by 9?
   a. 504
   b. 1519
   c. 6345
   d. 3692
   e. 123,456,789
2. Given a number not divisible by 9, sum the digits of that number. Keep summing the digits of your answer until you reach a single-digit number. This number is the remainder you get if the original number is divided by 9. Try this property on the following numbers:
   a. 1519
   b. 3692
   c. 1111
3. Any two-digit number minus the sum of its digits will be divisible by 9. Try this property using the following numbers:
   a. 19
   b. 35
   c. 93
   d. 27

   The above property of nine was illustrated in the table on page 34.

ACTIVITY PAGES for MAGIC NINE

See if you can figure out how the magician can predict the number of pennies you are holding in your fist and that you have eleven cents in your pocket.

MAGIC NINE TRICK

Place twenty pennies on a table in front of your subject. Now, turn your back to the table while you give directions 1–4 below to your subject.

1. Pick up a few pennies (any single-digit number) and put them in your pocket.
2. Count the number of pennies left on the table. Sum the digits in this number.
3. Pick up the number of pennies corresponding to this sum. Put them in your pocket, also.
4. Pick up some more pennies, but hold these pennies in a closed fist in front of you. Tell me when you are done.
5. When your subject is ready, turn and face him or her while you announce how many pennies are concealed in his or her fist and add, "You can keep the eleven cents in your pocket." This announcement is quite effective if you say it immediately upon turning to your subject.

OBSERVATIONS

1. How many choices does the subject have when selecting pennies in step 1 in the trick?

2. Pick a number to represent one of the choices above. Using this choice, follow the next two steps in the trick and determine how many pennies remain on the table at the end of step 3.

3. Repeat observation 2 for each of the other possible choices you listed in observation 1.

4. What can you conclude from the first three observations?

5. At the end of step 3 of the trick, how many pennies should the subject have in his or her pocket?

CONCLUSION

1. Use your observations above to explain how the magician is able to predict how many pennies are held in the subject's fist and how many are in the subject's pocket.

MAKING YOUR OWN TRICK

In the MAGIC NINE trick, twenty pennies were used. If more than 20 objects are used, a modification must be made in the trick. Try the trick as written but using 25 pennies. How does the trick fail? Try some other two-digit numbers greater than 20 to represent the original number of pennies. What happens in each case? Consider adding a new direction between step 3 and step 4 in the trick so the number of pennies left on the table will be nine. How many pennies would then be in the subject's pocket?

NUMBER 7 HIDDEN SUMS

TEACHER'S INSTRUCTIONS

After your subject hides two or three numbers in a chart (see page 43) by covering them with pennies, you will magically announce the sum of the hidden numbers.

SKILLS

- Discover number patterns
- Mentally compute sums for one- and two-digit numbers

MATERIALS

- Copy of the table (a reproducible blackline master is on page 43)
- A few pennies or markers

PERFORMING THE TRICK

Give the prepared table (see blackline master on page 43) and three pennies to your subject.

1. **Turn your back and ask your subject to cover two or three numbers on the chart.**
2. **When you turn to examine the chart, you will announce the sum of the hidden numbers.**
3. **Have your subject remove the pennies, one by one, and check to see if your sum is correct. A calculator may be used to compute this sum. Your prediction will match your subject's sum if you follow the directions below.**

HOW THE TRICK WORKS

From any number covered with a penny, count three numbers diagonally, or count six numbers horizontally (or vertically), and you will find a number which matches the hidden number. Try this in the table!

Since you can determine the hidden numbers by counting spaces in the table, you can mentally add these hidden numbers to determine your prediction.

When counting from the hidden numbers, be sure you do not give away the trick. Try to do your counting without pointing with your finger.

You may encounter a problem in this trick. Your subject may cover all of the entries for a given number. For example, there are only three 17s in the table and your subject might cover all three of these 17s. If the three 17s were covered, you would not be able to find any matching numbers by counting since there would be no 17s left. But, the table only contains the numbers from 1 to 17, so you will know that the three 17s are covered when you discover it is missing from the table. Therefore, all three of the missing numbers are 17s and the sum is 17 + 17 + 17 or 51.

AN ILLUSTRATION

Suppose your subject prepares the chart below by placing pennies on three numbers.

8	14	10	3	16	12	8	14
17	4	●	15	1	5	17	4
1	7	13	2	6	●	1	7
3	16	12	8	14	10	3	16
15	1	5	17	4	9	15	1
2	6	●	1	7	13	2	6
8	14	10	3	16	12	8	14

From the top-left penny, you can count 3 numbers diagonally and locate 9. Thus, one hidden number is a 9.

From the lower-left penny, you arrive at the third penny by counting diagonally, so the last two hidden numbers are equal. But, you cannot determine these numbers by counting from either penny. Checking the table for numbers from 1 to 17, you will notice that the number 11 is missing. Therefore, the last two numbers must be 11s, and the sum is 11 + 11 + 9 = 31.

When your subject uncovers the three hidden numbers and computes the sum, it will match your prediction.

WHY THE TRICK ALWAYS WORKS

The 7 × 8 table of numbers was constructed using the numbers from 1 to 17. Every time a number was written in the table, this same number was written in locations three spaces diagonally from that number and six spaces vertically or horizontally from that number. Therefore, if you remember this number pattern, you will be able to determine any number that might be covered in the chart. When you know the hidden numbers, you can mentally compute their sum.

USING THE TRICK

The table on page 43 can be duplicated for use with an individual or with a small group of students. If you perform the trick before a class, a transparency can be prepared for the overhead. Then when your back is turned, students can cover numbers using pennies or other suitable markers.

Notice that any number of pennies can be used, but you will have to determine the mental sum for all of the hidden numbers.

Students can discover how the magician makes the prediction by completing the activity pages (pp. 41–42). When your students know the secret behind the trick, encourage them to practice the trick and try it out on others.

Also, on activity page 42 is an exercise in which students design their own version of the HIDDEN SUMS trick.

HIDDEN SUMS and the MATHEMATICS CURRICULUM

Although the placement of numbers in the prepared table may appear random at first glance, it is not. The special number patterns are known to the magician but are difficult for the subject to discover in the two-dimensional array. Below are some exercises that concentrate on one- and two-dimensional number patterns.

1. Place numbers in the blanks to continue the number sequence.

   1, 2, 5, 10, 17, ___, ___, ___

2. A number sequence begins: 1, 4, 7, 10, 13. Circle the four numbers that continue the sequence.

   a. 15, 18, 21, 24

   b. 16, 19, 22, 25

   c. 14, 15, 16, 17

3. Find the missing numbers in the sequence of ordered pairs.

   (2, 3), (4, 6), (___, 9), (8, 12), (___, ___)

4. Use the hundreds table on page 50 to describe the pattern formed by each of the following:

   a. All the multiples of 10

   b. All the multiples of 5

   c. All the multiples of 2

   d. All the multiples of 3

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ACTIVITY PAGES for HIDDEN SUMS

This trick works because of the special nature of the table of numbers. See if you can discover what is so special about the table.

HIDDEN SUMS TRICK

Give the prepared table and three pennies to your subject.

1. Turn your back and ask your subject to cover two or three numbers on the chart.
2. When you turn to examine the chart, you will announce the sum of the hidden numbers.
3. Have your subject remove the pennies, one by one, and check to see if your sum is correct. A calculator may be used to compute this sum. Your prediction will match your subject's sum.

OBSERVATIONS

1. Locate all the 8s in the table. Describe how you can move from one 8 to the other 8s in the table.

2. Locate all the 14s in the table and, again, describe how you can move from one 14 to the others.

3. Repeat the observations above for other numbers in the table.

4. Make a list of all the counting numbers that appear in the table. Write these numbers in numerical order.

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CONCLUSION

1. Suppose someone covered a number in the table. Use your observations above to explain how you know which number was covered.

2. Suppose someone covered all the 10s in the table. Explain how you could determine that all the covered numbers were 10s.

MAKING YOUR OWN TRICK

Now that you know how this trick works, you can design your own trick. Make your own table of numbers. Make the table as large as you like, and use any numbers you wish. However, design the placement of the numbers in the table so that you will be able to determine the value of any hidden number. You can make your placement of numbers in any way you wish, but you will have to remember the pattern you used to arrange the numbers.

You can use any operation or operations that you wish for your prediction. But, again, remember that you will have to do this computation for your prediction. Now, write out the directions for performing your trick.

After you have practiced, try your trick on your family and friends.

8	14	10	3	16	12	8	14
17	4	9	15	1	5	17	4
1	7	13	2	6	11	1	7
3	16	12	8	14	10	3	16
15	1	5	17	4	9	15	1
2	6	11	1	7	13	2	6
8	14	10	3	16	12	8	14

NUMBER 8 PICK AND SUM

TEACHER'S INSTRUCTIONS

In the following trick, your subject picks five numbers from the table on page 49, but you will know the sum of those five numbers before the first number is selected.

SKILLS

- Add a column of two-digit numbers

MATERIALS

- A copy of the number table (a reproducible blackline master is on page 49)

PERFORMING THE TRICK

Give a copy of the number table to your subject. At this time, you can write your predicted sum on a piece of paper and seal it in an envelope. Let your subject hold this envelope until the end of the trick.

1. **Ask your subject to select any number from the first row in the table. This number can be circled or underlined.**
2. **Then ask your subject to select any number from the second row as long as it is not in the same column as the first number selected.**
3. **Ask your subject to repeat this procedure by selecting numbers from the third, fourth, and fifth rows, making sure that these numbers are not in the same column as any previously selected number.**
4. **After the five numbers have been selected, have your subject compute the sum of these numbers. Now, ask your subject to compare this sum with your prediction which is sealed in the envelope. They will match!**

HOW THE TRICK WORKS

The predicted sum is always 65, which means that you cannot repeat the trick too often and expect your subject to be surprised.

AN ILLUSTRATION

Suppose your subject has selected the numbers which are circled below.

1	2	3	4	⑤
6	7	⑧	9	10
11	12	13	⑭	15
⑯	17	18	19	20
21	㉒	23	24	25

The sum of these numbers (5 + 8 + 14 + 16 + 22) is 65, which matches your prediction.

WHY THE TRICK WORKS

There are exactly 120 possible selections of numbers that your subject can make in this trick, yet each of the sums is 65. Checking all 120 selections would be tedious. A simpler explanation of the prediction is shown below.

Consider the selection of numbers along the diagonal pictured below. Note that the SUM = 1 + 7 + 13 + 19 + 25 = 65.

①	2	3	4	5
6	⑦	8	9	10
11	12	⑬	14	15
16	17	18	⑲	20
21	22	23	24	㉕

Mark the numbers in the diagonal with counters of some sort—small scraps of paper could be used. Now, consider any one of the other 119 possible five-number selections that the subject might choose in the trick. For example, one such selection is 3, 10, 11, 17, and 24.

In row 1 of the table above, move the counter from 1 to the new selection, 3. This move adds 2 to the sum and SUM = 65 + 2 = 67. However, two numbers, 3 and 13, now appear in the same column. So, move the counter covering 13 to column 1 to cover the 11, which is another of the new selections. This move subtracts 2 from the sum, so the SUM = 67 – 2 = 65.

In row 2, move the counter from 7 to the new number 10. This move adds 3 to the sum, and SUM = 65 + 3 = 68. However, we now have 10 and 25 in the same column. So, move the counter on 25 to cover 24, which is another of the new selections. This move subtracts 1 from the sum, and the SUM = 68 – 1 = 67.

Now we have 24 and 19 in the same column. So, move the counter on 19 to cover 17, which is the last of the new selections. This last move subtracts 2 from the sum, and the SUM = 67 – 2 = 65, which is the same as the SUM for the original selection of numbers above.

USING THE TRICK

If you wish to perform this trick with a class, you can project the table of numbers using an overhead projector. Prepare the predicted sum, 65, to be revealed in any way you think suitable.

Have students take turns placing colored tiles on numbers they select from the five rows in the chart. However, make sure no two numbers selected are in the same column.

As these numbers are selected, have members of the class compute the sum. At the end of the trick, compare this sum with your prediction. The first few times you perform the trick, the result is quite surprising, but quickly, all students will come to realize that the sum is always 65. At this point, you may wish to explore why this is true. This can be done as a classroom activity or by having students complete the activity pages (pp. 47–48) individually or in small groups.

On activity page 48, constructing variations of this trick is suggested as a student exercise.

PICK AND SUM and the MATHEMATICS CURRICULUM

The hundreds table (see page 50) is regularly used in the mathematics classroom when discussing number patterns, multiples, and factors of numbers. Some example activities are listed below.

1. Circle all the multiples of 5. Describe the pattern formed by these numbers.
2. Circle all numbers that are divisible by 3. Describe the pattern formed by these numbers.

It is not necessary that the hundreds table be used in the investigations above. The same questions could be asked using any rectangular array of counting numbers, such as the number table used in PICK AND SUM.

The hundreds table is also used in the curriculum to discuss prime and composite numbers. The Sieve of Eratosthenes is a method by which all nonprimes are deleted from the hundreds table, leaving the set of primes less than 100.

ACTIVITY PAGES for PICK AND SUM

You may have guessed that the sum of any five numbers chosen in this trick is 65. See if you can discover why this is true.

PICK AND SUM TRICK

Give a copy of the number table to your subject. At this time, you can write your predicted sum on a piece of paper and seal it in an envelope. Let your subject hold this envelope until the end of the trick.

1. **Ask your subject to select any number from the first row in the table. This number can be circled or underlined.**
2. **Then ask your subject to select any number from the second row as long as it is not in the same column as the first number selected.**
3. **Ask your subject to repeat this procedure by selecting numbers from the third, fourth, and fifth rows, making sure that these numbers are not in the same column as any previously selected number.**
4. **After the five numbers have been selected, have your subject compute the sum of these numbers. Now, ask your subject to compare this sum with your prediction which is sealed in the envelope. They will match!**

OBSERVATIONS

Use a copy of the number table and five pennies or other counters in answering the observation questions below.

1. Use pennies to cover the five numbers along the diagonal: 1, 7, 13, 19, and 25. What is the sum of these five numbers?

2. Choose five different numbers that a subject might select in this trick. List these numbers in order of size, smallest to largest.

3. Move the penny covering the 1 to the new number selection in the first row. How does this move change the sum of the five covered numbers? Write the new sum.

4. Move the penny covering the 7 to the next number in the new list. Does this move increase or decrease the sum found in 3 above? By how much? What is the new sum?

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5. Move the penny covering the 13 to the next new number. How does this move affect the sum found in 4 above? Write the new sum.

6. Move the penny covering the 19 to the next new number. How does this move affect the sum found in 5 above? Write the new sum.

7. Finally, move the penny covering the 25 to the last number in the new list. How does this move affect the sum found in 6 above? Write the new sum.

CONCLUSION

1. What is the overall effect on the sum when you move the five pennies from their diagonal positions to a new selection of numbers?

2. Explain why the sum of any five numbers selected by a subject in this trick is 65.

MAKING YOUR OWN TRICK

The explanation above suggests a procedure which could be used to modify the original trick. For this trick, use the hundreds table (p. 50). Using this table, a subject can make 3,628,800 different choices of numbers in the trick.

USING THE RULES FOR PICK AND SUM, how many numbers would the subject select in this trick? What would your prediction be in this trick? How could you convince someone that each of the 3,628,800 possible selections leads to this same sum?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	343	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

NUMBER 9 DICE ARITHMETIC

TEACHER'S INSTRUCTIONS

While your back is turned, a subject rolls three dice and then rerolls one of them. After computing sums of numbers rolled on these unseen dice, you turn and roll the dice yourself. At this point, you can announce your subject's sum.

SKILLS

- Addition of one- and two-digit numbers
- Multiplication facts for 7

MATERIALS

- Paper and pencil
- Three standard dice

PERFORMING THE TRICK

Hand three dice to your subject and explain that some simple arithmetic will be carried out as the dice are rolled. Have paper and pencil at hand so your subject can write the final result.

Now, turn your back to your subject and give directions 1–7 below.

1. **Roll the three dice.**
2. **Sum the numbers on the three dice.**
3. **Pick up one of the dice and note the number on the bottom.**
4. **Add this number to the previous sum.**
5. **Roll the die that you are holding.**
6. **Add the new roll to the last sum.**
7. **Write the final sum on a piece of paper, and place it facedown on the table.**
8. **At this point, turn around and pick up the dice. Immediately, you will know what sum has been written by your subject. To distract your subject while you make your prediction, roll the three dice. Make it appear that you are using this roll to make your prediction. Magicians call this misdirection. After you make your prediction, have your subject turn over the paper to confirm that your prediction is correct.**

HOW THE TRICK WORKS

After your subject has completed the seven steps above, turn and pick up the dice. As you pick them up, note the sum on the three dice. Add seven to this sum, and that is your subject's hidden sum.

Now roll the dice, and appear to be thinking as you study the dice. At any time, you can announce your prediction, and it will match the hidden sum.

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AN ILLUSTRATION

While your back is turned, suppose your subject has rolled the dice as shown below.

$\boxed{3}\ \boxed{5}\ \boxed{1}$

The table below follows the subject through the seven steps in the trick.

STEP	ACTIVITY	COMPUTATION
1	roll the 3 dice	assume a 3, 5, and 1 are rolled
2	sum the numbers	3 + 5 + 1 = 9
3	pick up a die & note bottom number	say 3 is picked up, 4 is on the bottom
4	add this number to the sum	9 + 4 = 13
5	reroll die	say a 2 is rolled
6	add this to the sum	13 + 2 = 15
7	write and hide answer	15 is written

When you turn to pick up the three dice, you will see what is pictured below.

$\boxed{2}\ \boxed{5}\ \boxed{1}$

For your prediction, add the three numbers above (2 + 5 + 1 = 8) and add seven to the sum (8 + 7 = 15). The answer, 15, is your subject's hidden sum.

If your subject's sum is less than 19, here is a clever way to announce your prediction: roll the dice until the sum matches your subject's. For example, suppose you roll a 6, 4, and 1. Pick up the 1 and reroll it until it comes up 5. Now your dice add up to 15 (6 + 4 + 5 = 15), which matches the hidden sum. If your subject's sum is 19 or more, you can devise other ways to make your prediction seem to come from rolling the dice.

WHY THE TRICK WORKS

The secret of this trick is that the sum of the numbers on the top and on the bottom of any die is always seven. Pick up a die and check this.

Let us follow the illustration above, but this time we will not sum the numbers until the last direction has been given.

STEP	DICE ROLLED	SUBJECT'S COMPUTATION
1	3 5 1	
2	3 5 1	3 + 5 + 1
3	__ 5 1	4 on bottom
4	__ 5 1	(4 + 3) + 5 + 1 = (7) + 5 + 1
5	2 5 1	assume a 2 is rolled
6	2 5 1	(7) + 2 + 5 + 1
7	2 5 1	write sum, 15

When you turn to the table, you see the three dice listed in step 7 above, namely, 2, 5, and 1. When you add a 7 to this sum, you have the subject's last computation shown in step 6 above. These two sums will always match.

USING THE TRICK

It is difficult for the subject to realize how you are making your prediction, especially if you "misdirect" the subject. Therefore, this trick can be repeated many times with the same subjects, and the trick will still amaze them. When students discover that the top and bottom of a die add up to seven, this might give them a clue to your prediction.

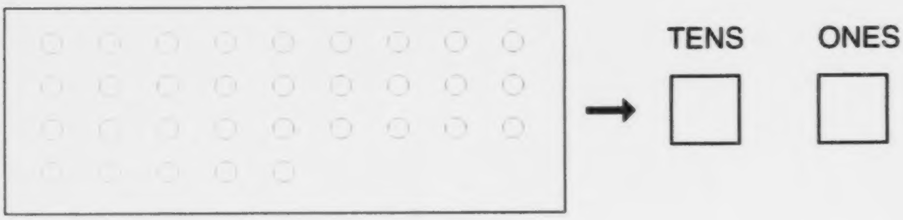
Students can discover the "trick" if they follow the observations on the activity pages (pp. 55–57). Once a student learns the prediction rule, this knowledge can be applied to designing other prediction tricks using dice.

DICE ARITHMETIC and the MATHEMATICS CURRICULUM

For students to perform or create their own versions of the trick, they must be able to compute mental sums and know their "seven times table."

Below are some sample exercises from the mathematics curriculum that address these skills.

1. Draw a ring around each group of ten circles. Write the number of tens and ones in the boxes.



2. Rewrite the numbers so the sum in the parentheses is ten. Write the sum in the box.

   $7 + 6 + 3 = (\_\_\_\_ + \_\_\_\_) + \_\_\_\_ = \boxed{}$

3. Fill in the blank.

   $2 + 6 + \_\_\_\_ = 10$

4. Fill in the blanks.

   a. $7 \times 3 = \_\_\_\_$

   b. $7 \times \_\_\_\_ = 42$

ACTIVITY PAGES for DICE ARITHMETIC

Since the subject controls the dice out of the magician's view, the prediction seems like "magic." See if you can discover the magician's secret.

DICE ARITHMETIC TRICK

Hand three dice to your subject and explain that some simple arithmetic will be carried out as the dice are rolled. Have paper and pencil at hand so your subject can write the final result.

Now, turn your back to your subject and give directions 1–7 below.

1. Roll the three dice.
2. Sum the numbers on the three dice.
3. Pick up one of the dice and note the number on the bottom.
4. Add this number to the previous sum.
5. Roll the die that you are holding.
6. Add the new roll to the last sum.
7. Write the final sum on a piece of paper, and place it facedown on the table.
8. At this point, turn around and pick up the dice. Immediately, you will know what sum has been written by your subject. To distract your subject while you make your prediction, roll the three dice. Make it appear that you are using this roll to make your prediction. Magicians call this misdirection. After you make your prediction, have your subject turn over the paper to confirm that your prediction is correct.

OBSERVATIONS

Roll three dice and place them in a row. Then fill in the blanks in the observations below, using the numbers appearing on these dice.

1. Write the numbers appearing on the three dice.

   die 1 = _____ die 2 = _____ die 3 = _____

2. In step 2 of the trick, write the indicated sum.

   sum = ______ + ______ + ______

   die 1 die 2 die 3

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3. Pick up die 1 and note the bottom number.

bottom number = _____

4. In step 4 of the trick, add the bottom number to the previous sum.

sum = (_____ + _____) + _____ + _____
bottom die 1 die 2 die 3

What is the sum of the numbers in parentheses? Will this sum remain the same for any number that might appear on die 1? Rewrite the sum above after adding the numbers in parentheses.

sum = (_____) + _____ + _____
die 2 die 3

5. Reroll the die that you picked up.

number rolled = _____

6. Add this new number to the last sum.

sum = (_____) + _____ + _____ + _____
new roll die 2 die 3

7. When the magician turns to pick up the dice, what numbers are showing on the dice?

die 1 = _____ die 2 = _____ die 3 = _____

8. Sum the numbers in observation 7.

sum = _____ + _____ + _____

Now, add 7 to this sum.

sum = 7 + _____ + _____ + _____

Compare this new sum to the sum you found in observation 6 above. What do you observe?

CONCLUSION

1. Use the results of your observations above to explain how the magician can always predict the subject's hidden number.

MAKING YOUR OWN TRICK

You discovered in DICE ARITHMETIC that the sum of the numbers on the top and bottom of any die is seven. Using this fact and your knowledge of the "seven times table," devise a new prediction trick.

From a pile of dice, ask your subject to choose some dice and roll them. Have the subject add the numbers on the tops of the dice, turn over the dice, and add the numbers on the bottoms of the dice to the previous sum.

You can predict the final sum as soon as your subject picks up the dice. Work out a prediction rule. What does the prediction rule have to do with the "seven times table"? After you practice, find a subject and try your trick.

You may wish to try creating a trick using polyhedra dice. See page 62 from the trick EXPERIMENTAL DICE.

NUMBER 10 EXPERIMENTAL DICE

TEACHER'S INSTRUCTIONS

In this trick, your subjects conduct experiments using a pair of dice. A flowchart which is included in the activity (from *Mathematical Reasoning Book 2*, Critical Thinking Press & Software, p. 165) gives specific instructions for conducting the experiment.

SKILLS

- Read a flowchart
- Multiply single-digit numbers
- Add a column of two-digit numbers

MATERIALS

- A pair of dice (preferably two different colors)
- Copy of the flowchart (a reproducible blackline master is on page 63)
- Polyhedra dice for follow-up activities on page 62

PERFORMING THE TRICK

1. **Choose a volunteer and give him or her a pair of six-sided dice.**
2. **Designate one of the dice as DIE 1 and the other as DIE 2.**
3. **Instruct the volunteer to roll the dice but not to show you what he or she rolls.**
4. **Have the volunteer find the following products:**

   (top of DIE 1) × (top of DIE 2)
   (bottom of DIE 1) × (bottom of DIE 2)
   (top of DIE 1) × (bottom of DIE 2)
   (top of DIE 2) × (bottom of DIE 1)
5. **Write the number 49 on a piece of paper and give it to a different person to hold. Announce: Without seeing what you rolled, I have written the sum of these four products on a piece of paper.**
6. **Have the volunteer add the four products then have him or her say the sum aloud.**
7. **Have the person holding your prediction show the paper on which you have written 49.**

HOW THE TRICK WORKS

If you use a standard pair of dice and follow the instructions carefully, the sum will always be 49.

AN ILLUSTRATION

Refer to the flowchart on page 63. If one rolls a 6 and a 4 with the pair of dice, the verification that the sum is 49 is shown below.

Top of die 1, A = 6

Bottom of die 1, B = 1

Top of die 2, C = 4

Bottom of die 2, D = 3

A × D = 18

C × B = 4

A × C = 24

B × D = 3

SUM, S = 49

WHY THE TRICK ALWAYS WORKS

This trick works because of the way in which numbers are placed on a standard die. The numbers are arranged so that the sum of the numbers on opposites sides is always 7. There are six possible outcomes for die 1 and six possible outcomes for die 2. Therefore, there are 6 × 6 (or 36) possible rolls for a pair of dice.

To verify that the sum is always 49, one could verify that each of the 36 possible rolls leads to that sum. To do this, students can divide up the task. With a group of eighteen students, each student need only verify two of the 36 possible rolls.

USING THE TRICK

After demonstrating the trick, have your students work individually or in groups, following the instructions on the flowchart found on page 63. The flowchart can be duplicated and distributed as needed. Each individual or group will also need a pair of dice; dice of two different colors would be helpful but are not necessary. Alternatively, the trick can be done as a teacher-led classroom activity. In this case, one pair of dice and a transparency of the flowchart can be used.

After the students have completed several rolls of the dice, ask them if they believe that the sum produced by following the instructions on the flowchart will always be 49. The students should have enough evidence to conclude inductively that the sum will always be 49.

Verify this conjecture by using questions such as the following:

- How many possible outcomes are there when you roll a pair of dice?
- If we filled in the blanks on the flowchart for each of these possible outcomes, would this verify that the sum is always 49?
- How can you be sure that the sum is always 49?

After students inductively conclude that the sum will always be 49, you can lead the class through a discovery of this fact or you can have the students work cooperatively using the activity pages (pp. 61–62).

Encourage students to perform the trick using a pair of dice but without referring to the flowchart. Once students have learned the procedure for the trick, they can then ask friends and family members to roll the dice and perform the calculations, and the students can always predict the outcome, namely, 49.

Since the answer is always 49, this trick loses its impact when repeated with the same subject. So, in a suggested exercise on page 62, alterations are made in the trick so the sum is not always 49. For this extending activity, polyhedra dice are required. These dice can be found in most catalogues of companies that distribute supplementary mathematics materials. Along with the standard die with six sides (hexahedron), look for the octahedron (8 sides), the dodecahedron (12 sides), and the icosahedron (20 sides).

EXPERIMENTAL DICE and the MATHEMATICS CURRICULUM

Below are some exercises that you can do on the board with your class that address the concepts underlying this trick.

1. The six numbers appearing on a standard die are 1 2 3 4 5 6. Draw lines connecting the pairs of numbers which appear opposite each other on a die.

2. Three other types of dice and the numbers appearing on each are listed below. For each die, draw lines connecting the pairs of numbers that you think appear on opposite sides of the die.

   Octahedron: 1 2 3 4 5 6 7 8

   Dodecahedron: 1 2 3 4 5 6 7 8 9 10 11 12

   Icosahedron: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

3. Suppose a pair of ordinary dice are rolled. How many outcomes are there for the first die? For the second die? In how many ways can the pair of dice land when rolled?

4. Answer the questions in 3 above for a pair of octahedra dice. Do the same for a pair of dodecahedra dice and then icosahedra dice.

ACTIVITY PAGES for EXPERIMENTAL DICE

Use the flowchart to help you understand this trick.

EXPERIMENTAL DICE TRICK

1. Roll a pair of six-sided dice.
2. Find the following products:

   (top of DIE 1) × (top of DIE 2)

   (bottom of DIE 1) × (bottom of DIE 2)

   (top of DIE 1) × (bottom of DIE 2)

   (top of DIE 2) × (bottom of DIE 1)
3. Find the sum of the products.
4. You can predict the sum of these four products before the dice are rolled.

OBSERVATIONS

1. How many possible outcomes are there for the first die rolled? List them.

2. How many possible outcomes are there for the second die rolled? List them.

3. How many different rolls are there for the pair of dice? List them as ordered pairs. The pair (3, 4) means a 3 came up on the first die and a 4 came up on the second die.

4. With your classmates, divide up the task of working through the flowchart for each of the possible rolls listed in 3 above. What is the sum of the four products in each case?

CONCLUSION

1. Is the sum always 49, no matter what two numbers appear on the dice?

2. Write a brief explanation stating why you are sure that the sum is always 49.

MAKING YOUR OWN TRICK

Standard dice have six sides (hexahedron), but some dice have more sides: the octahedron (8 sides), the dodecahedron (12 sides), and the icosahedron (20 sides).

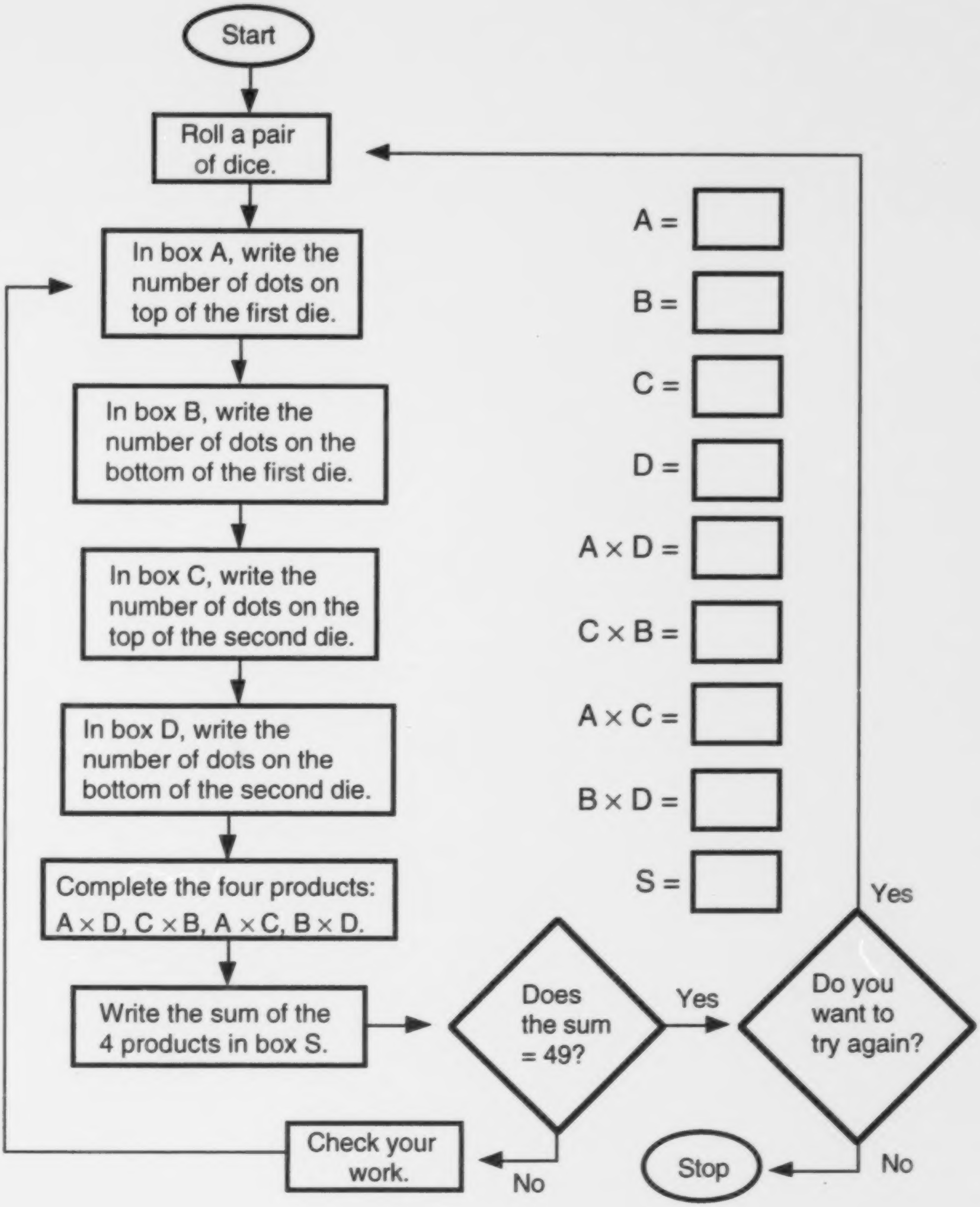
On a hexahedron, or normal six-sided die, the sum of the numbers on opposite sides of the die is always a constant, namely, 7. Check a hexahedron to verify this. On some commercially sold polyhedra dice, the sums on the opposite sides are also constants: 9 for the octahedron, 13 for the dodecahedron, and 21 for the icosahedron. But be careful because some companies do not adhere to this rule, and the dice trick will not work unless this sum is constant.

Now, follow the steps in the flowchart for a pair of octahedra. What sum do you compute at the end of the trick? Do the same for a pair of dodecahedra and for a pair of icosahedra. What are these sums? What would happen if you were to pick up one hexahedron and one dodecahedron and conduct the experiment? How about other dice combinations?

After you have worked out the possible sums for different dice combinations, write these sums down in a chart for reference.

Now, if you find a suitable set of polyhedra dice and have worked out a reference chart, you can make up a very interesting prediction trick.

From your collection of different polyhedra dice, ask your subject to select any two dice. Have your subject roll the pair of dice and do the computations suggested in the flowchart. Of course, as soon as the dice are selected, you will be able to predict the final sum by referring to your reference chart. This prediction can be written on a card and placed facedown on the table. When the subject has completed the trick, your prediction card can be turned over. With a little practice, your prediction will always be correct.


Start
Roll a pair of dice.
In box A, write the number of dots on top of the first die.
In box B, write the number of dots on the bottom of the first die.
In box C, write the number of dots on the top of the second die.
In box D, write the number of dots on the bottom of the second die.
Complete the four products: A × D, C × B, A × C, B × D.
Write the sum of the 4 products in box S.
Does the sum = 49?
Yes
No
Check your work.
Do you want to try again?
Yes
No
Stop
A =
B =
C =
D =
A × D =
C × B =
A × C =
B × D =
S =

NUMBER 11 THE MONEY TRANSFER

TEACHER'S INSTRUCTIONS

In the following trick, your subject arranges piles of coins on a table and then transfers or removes coins from these piles. Without ever seeing the original piles of coins or observing the transfers, you will predict the number of coins remaining on the table.

SKILLS

- Multiply a single-digit number by 3
- Memorize a set of six directions

MATERIALS

- A container with 50 to 100 pennies

PERFORMING THE TRICK

Give your subject the container of pennies. For the remainder of the trick turn your back so you cannot see what the subject is doing while you give directions 1–7 below.

1. Ask your subject to make three piles of pennies on the table so that each pile has the same number of pennies. Have the piles arranged so that there is a left pile, a center pile, and a right pile. It is not necessary to use all the pennies. Unused pennies should be returned to the container.
2. Ask someone else in the room to suggest a small, positive whole number. This small, positive whole number will be referred to as *N* in the directions that follow.
3. Ask your subject to transfer *N* pennies from the left pile to the center pile. If this is impossible to do, then ask for a smaller, positive whole number so that the transfer can take place.
4. After the first transfer, have your subject transfer *N* pennies from the right pile to the center pile.
5. Ask your subject to pick up all the pennies remaining in the left pile and return them to the container.
6. Now, ask your subject to remove from the center pile as many pennies as there are in the right pile and place these pennies in the container.
7. Finally, have your subject pick up all the pennies in the right pile and return them to the container.
8. At this point in the trick, you can predict the number of pennies remaining on the table. Your prediction will always work if you remember the one simple rule below.

HOW THE TRICK WORKS

Multiply the small, positive whole number (suggested by someone during the trick) by 3. This is your prediction.

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AN ILLUSTRATION

Suppose your subject starts with 30 pennies in each of the three piles. Also, suppose someone else has suggested 6 as the small, positive whole number. After the first transfer of pennies (step 3), the left, center, and right piles will have 24, 36, and 30 pennies, respectively.

After the second transfer (step 4), the piles will have 24, 42, and 24 pennies. At this point, the left pile is removed.

The next transfer (step 6) removes 24 pennies from the center pile, leaving 18 in the center. When the right pile is removed, there will be 18 pennies left on the table.

Note that your prediction, $3 \times 6 = 18$, matches the number of pennies on the table.

WHY THE TRICK WORKS

This trick can be explained by choosing one number to represent the number of pennies placed in each pile and another number to represent the small, positive whole number. Note that this small, positive whole number must be less than the number of pennies in each of the piles.

For each step in the trick, we can determine the number of pennies in the piles after each transfer is completed. For example, suppose your subject places 12 pennies in each of the three piles and the small, positive whole number is 5. The table below keeps track of the pennies in each of the three piles.

NUMBER OF PENNIES IN THE THREE PILES

STEP IN THE TRICK	PILE 1	PILE 2	PILE 3
1	12	12	12
3	$12 - 5$	$12 + 5$	12
4	$12 - 5$	$12 + 5 + 5$	$12 - 5$
5	0	$12 + 5 + 5$	$12 - 5$
6	0	$12 + 5 + 5 - (12 - 5)$	$12 - 5$
7	0	$12 + 5 + 5 - (12 - 5)$	0

Thus, the number of pennies left in the center pile is

$$\begin{aligned} & 12 + 5 + 5 - (12 - 5) \\ &= 5 + 5 + 12 - (7) \\ &= 5 + 5 + (12 - 7) \\ &= 5 + 5 + 5 \\ &= 3 \times 5 \end{aligned}$$

The final equation (3×5) is 3 times the small, positive whole number (5) that was suggested in step 2 of the trick.

This argument can be repeated with the number 12 replaced by any number of pennies and with 5 replaced by any small, positive whole number, as long as that number is less than or equal to the number of pennies in each of the three piles.

USING THE TRICK

After performing the trick and making your first prediction, you may decide to have your students discover how the trick works. This can be done as a classroom activity where you lead students through an inductive discovery process. Each time you repeat the trick, you can alter the number of pennies placed in each pile. After that you can keep the number in each pile constant and alter the value of the small, positive whole number.

Alternatively, you may wish to have students work individually or in groups using the activity pages (pp. 67–68) to discover how the predictions are made.

Also found on activity page 68 is an exercise in which students extend the trick by using coins of different denominations.

THE MONEY TRANSFER and the MATHEMATICS CURRICULUM

Central to understanding this trick is translating verbal or written phrases into expressions using mathematical operations or into physical models. Here are a few examples from the elementary mathematics curriculum.

WRITTEN PHRASE	EXPRESSION WITH OPERATIONS
four more that twelve	12 + 4
remove 6 from a set of 21	21 – 6
increase 12 by 5 and again by 5	12 + 5 + 5

WRITTEN PHRASE	PHYSICAL MODEL
remove 4 from a set of 9 elements	■ ■ ■ ■ ■ (■ ■ ■ ■)

WRITTEN PHRASE	USING VARIABLES AND OPERATIONS
remove N from a set of X elements	X – N

The performance of the trick illustrates the use of a physical model for the verbal steps in the trick. The table on page 65 illustrates using expressions with operations, and if variables are used in the table on page 65, you will have an algebraic verification of how the prediction works.

ACTIVITY PAGES for THE MONEY TRANSFER

Although the magician does not have knowledge or control over the number of pennies used or the number of pennies transferred between piles, the magician's prediction is always correct. See if you can discover why.

THE MONEY TRANSFER TRICK

1. Ask your subject to make three piles of pennies on the table so that each pile has the same number of pennies. Have the piles arranged so that there is a left pile, a center pile, and a right pile. It is not necessary to use all the pennies. Unused pennies should be returned to the container.
2. Ask someone else in the room to suggest a small, positive whole number. This small, positive whole number will be referred to as *N* in the directions that follow.
3. Ask your subject to transfer *N* pennies from the left pile to the center pile. If this is impossible to do, then ask for a smaller, positive whole number so that the transfer can take place.
4. After the first transfer, have your subject transfer *N* pennies from the right pile to the center pile.
5. Ask your subject pick up all the pennies remaining in the left pile and return them to the container.
6. Now, ask your subject to remove from the center pile as many pennies as there are in the right pile and place these pennies in the container.
7. Finally, have your subject pick up all the pennies in the right pile and return them to the container.
8. At this point in the trick, you can predict the number of pennies remaining on the table.

OBSERVATIONS

Suppose at the start of the trick there are 10 pennies in each pile. Also, suppose someone has suggested 4 as the small, positive whole number.

1. After step 3 in the trick, how many additional pennies are there in the center pile?
2. After step 4, how many additional pennies are there in the center pile? How many are there in the right pile?

3. Does step 5 change the number of pennies in the center pile?

4. From the original number of pennies in the center pile, subtract the number remaining in the right pile. What is this difference?

5. Add the answer found in observation 4 above to the number of pennies that was transferred to the center pile in observations 1 and 2 above. What is your answer?

6. When the right pile is removed in step 5 of the trick, how many pennies remain in the center pile?

CONCLUSION

1. Explain how the magician can predict the number of pennies remaining on the table by just knowing the small, positive whole number that was suggested during the trick.

2. Explain what happens if that small, positive whole number is greater than the number of pennies in each of the original piles.

MAKING YOUR OWN TRICK

Design your own money transfer trick. Instead of just pennies, try using a container of pennies, nickels, and dimes. Have your subject make sure that each pile contains the same amount of money (for example, each pile might contain coins that add up to 50¢). Instead of having three piles of coins you may design your trick to have five piles or any other odd number (you need to have a middle pile).

Now, outline the steps in a trick that will transfer a suggested amount of money from various piles to the center pile or to the container. At the end of the trick, determine how you can predict the amount of money left on the table.

NUMBER 12 LET THE CARDS PREDICT

TEACHER'S INSTRUCTIONS

In many magic tricks, a magician predicts a number thought of by a subject. In this trick, the set of prepared cards shown below (and on page 75) is used to predict the number.

SKILLS

- Place value and face value
- Expanded notation (base ten/base two)
- Mental computation of sums

MATERIALS

- A copy of the five cards (a reproducible blackline master is on page 75)

PERFORMING THE TRICK

1. Ask your subject to think of any number from 1 to 30. This number could be the person's age or any number that you are sure will be 30 or smaller.
2. Now, hand your subject the five prepared cards. Ask your subject to examine the five cards and hand you each card on which the selected number appears.
3. As soon as you receive the last card, you can announce the number thought of by your subject.

HOW THE TRICK WORKS

As the cards are handed to you, mentally add the numbers appearing in the upper left-hand corner of each card. For example, if the first four cards shown below were handed to you, then the subject's number is 1 + 2 + 4 + 8, or 15. With a little practice at mental arithmetic, you will immediately reveal the correct number.

Card 1

1	3	5	7	9
11	13	15	17	19
21	23	25	27	29

Card 2

2	3	6	7	10
11	14	15	18	19
22	23	26	27	30

Card 3

4	5	6	7	12
13	14	15	20	21
22	23	28	29	30

Card 4

8	9	10	11	12
13	14	15	24	25
26	27	28	29	30

Card 5

16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

To make your prediction a mystery and to distract your subject from guessing what you are doing, it is very effective to carry on a conversation with your subject during the trick.

AN ILLUSTRATION

Suppose you ask your subject to think of a whole number which estimates the number of hours the subject watches TV during the week. Let us assume your subject is a frequent viewer and estimates 18 hours per week. After examining the five cards, the subject should hand you card 2 and card 5 since these are the only cards on which an 18 appears.

As soon as you see the two cards, you add 2 + 16 = 18 and announce your prediction, "Eighteen hours per week! Isn't that a lot of TV?"

WHY THE TRICK ALWAYS WORKS

Your prediction is based on the fact that any number from 1 to 31 (31 was left off the cards so that there would be fifteen numbers per card) can be written in a unique way as a sum of numbers selected from the following set: 1, 2, 4, 8, and 16. Note that these numbers appear in the upper left-hand corners of the cards.

Some examples of using 1, 2, 4, 8, and 16 to represent other numbers are shown below.

5 = 1 + 4, so the number 5 appears on the 1 and 4 cards

9 = 1 + 8, so the number 9 appears on the 1 and 8 cards

12 = 4 + 8, so the number 12 appears on the 4 and 8 cards

13 = 1 + 4 + 8, so 13 appears on the 1, 4, and 8 cards

22 = 2 + 4 + 16, so 22 appears on the 2, 4, and 16 cards

27 = 1 + 2 + 8 + 16, so 27 appears on the 1, 2, 8, and 16 cards

Thus, if your subject chooses 29, the cards you will be given are the 1, 4, 8, and 16 cards since 29 = 1 + 4 + 8 + 16. By adding these four numbers (found in the upper left-hand corners of the cards), you will know the number thought of, namely, 29.

Note that since 31 = 1 + 2 + 4 + 8 + 16, it could have been included in the trick and would appear on each of the cards. The number 31 can be written in expanded notation in base two as follows: $31 = (1 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + 1$ or in standard base two notation as $31 = 11111_{\text{two}}$.

USING THE TRICK

If the trick is performed before a class, the blackline master on page 75 can be used to make a transparency. Subjects can simply announce on which of the cards their number appears. Your prediction can be made instantaneously.

On the activity pages (pp. 72–74), students are led through a discovery of how mathematics is used to design the cards and, thus, how the prediction can be made.

Also, on activity page 74, suggestions are made for extending the trick by preparing a new set of cards.

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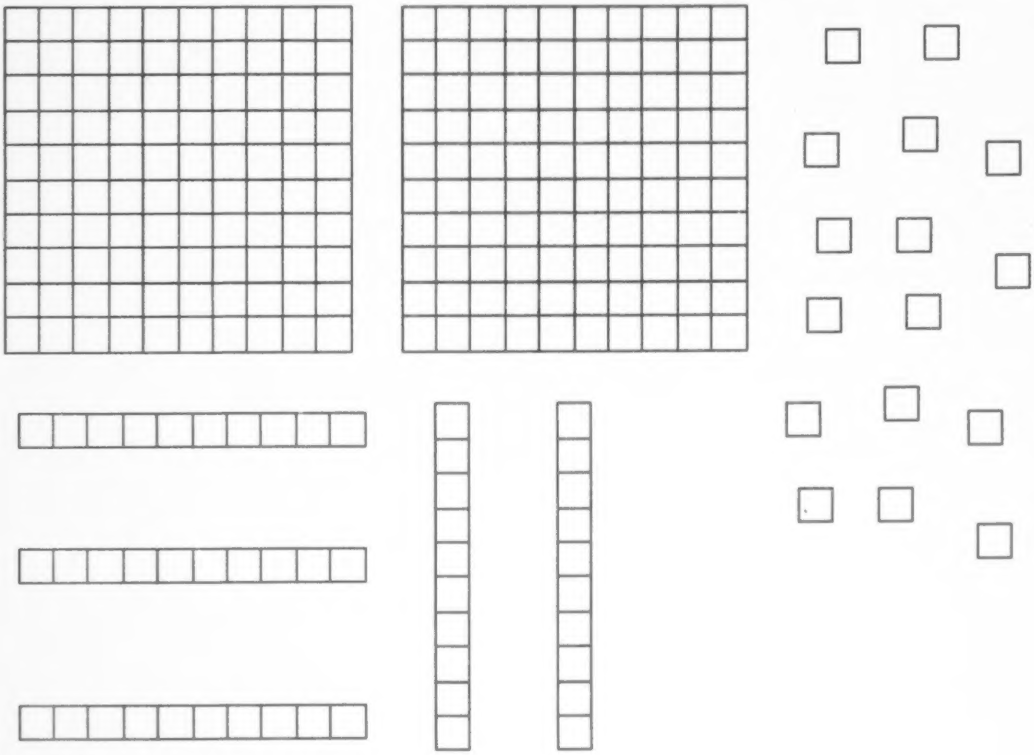
LET THE CARDS PREDICT and the MATHEMATICS CURRICULUM

The mathematics used in this trick deals with place value and face value. The cards are designed by first expressing the numbers from 1 to 30 in expanded base two notation and then placing the numbers on the appropriate cards. Of course, it is not necessary for the magician to know binary notation or how the cards were constructed in order to perform the trick or make the correct prediction.

Below are some examples of place value and face value from the elementary mathematics curriculum.

1. Place a single digit in each blank to make the computation match the given set below. In the box, write the number in standard notation.

   (_____ × 100) + (_____ × 10) + _____ = ☐



2. Place a single digit in each blank to make the statement below true.

   (_____ × 10^2) + (_____ × 10^1) + _____ = 880

3. Place either a 0 or 1 in each of the blanks to make the following statement true:

   30 = (_____ × 2^4) + (_____ × 2^3) + (_____ × 2^2) + (_____ × 2^1) + _____

4. Write the number 30 in base two notation, or binary notation.

ACTIVITY PAGES for LET THE CARDS PREDICT

The key to this trick is the special way in which the cards are designed. When you know how they were prepared, you will know how and why the magician's prediction works.

LET THE CARDS PREDICT TRICK

1. Ask your subject to think of any number from 1 to 30. This number could be the person's age or any number that you are sure will be 30 or smaller.
2. Now, hand your subject the five prepared cards. Ask your subject to examine the five cards and hand you each card on which the selected number appears.
3. As soon as you receive the last card, you can announce the number thought of by your subject.

OBSERVATIONS

Use the cards below to answer the observation questions that follow.

Card 1

1	3	5	7	9
11	13	15	17	19
21	23	25	27	29

Card 2

2	3	6	7	10
11	14	15	18	19
22	23	26	27	30

Card 3

4	5	6	7	12
13	14	15	20	21
22	23	28	29	30

Card 4

8	9	10	11	12
13	14	15	24	25
26	27	28	29	30

Card 5

16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

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In each blank below, place the digit 0 or 1 to make the statement true.

1. 5 = (___ × 16) + (___ × 8) + (___ × 4) + (___ × 2) + ___

   List all the cards on which 5 appears. List the numbers appearing in the upper left-hand corners of the cards you listed. What is the sum of these numbers?

2. 9 = (___ × 16) + (___ × 8) + (___ × 4) + (___ × 2) + ___

   List all the cards on which 9 appears. List the numbers appearing in the upper left-hand corners of the cards you listed. What is the sum of these numbers?

3. 13 = (___ × 16) + (___ × 8) + (___ × 4) + (___ × 2) + ___

   List all the cards on which 13 appears. List the numbers appearing in the upper left-hand corners of the cards you listed. What is the sum of these numbers?

4. 26 = (___ × 16) + (___ × 8) + (___ × 4) + (___ × 2) + ___

   List all the cards on which 26 appears. List the numbers appearing in the upper left-hand corners of the cards you listed. What is the sum of these numbers?

5. 27 = (___ × 16) + (___ × 8) + (___ × 4) + (___ × 2) + ___

   List all the cards on which 27 appears. List the numbers appearing in the upper left-hand corners of the cards you listed. What is the sum of these numbers?

6. Write any other number from 1 to 30. Place this number in the first blank below and then complete the equation.

   ___ = (___ × 16) + (___ × 8) + (___ × 4) + (___ × 2) + ___

   List all the cards on which the number you selected appears. List the numbers appearing in the upper left-hand corners of the cards you listed. What is the sum of these numbers?

CONCLUSION

1. Use your observations above to explain on which card any number (from 1 to 30) should appear.

2. Explain how a magician can make a prediction when handed cards by a subject.

MAKING YOUR OWN TRICK

Any number from 1 to 63 can be written uniquely as a sum of numbers selected from the following set: 1, 2, 4, 8, 16, and 32. For example, 42 = 2 + 8 + 32 and 63 = 1 + 2 + 4 + 8 + 16 + 32. Make a new set of cards (six in this case) for your trick. Start by listing all the numbers from 31 to 63 and writing each as a sum of numbers selected from the set 1, 2, 4, 8, 16, and 32, as shown above. Then place each number from 1 to 63 on the appropriate cards. For example, since 43 = 1 + 2 + 8 + 32, it should appear on the 1, 2, 8, and 32 cards. When your six new cards are completed, try out your new trick. Ask your subject to pick any number from 1 to 63 and follow the procedure from LET THE CARDS PREDICT.

Card 1

1	3	5	7	9
11	13	15	17	19
21	23	25	27	29

Card 2

2	3	6	7	10
11	14	15	18	19
22	23	26	27	30

Card 3

4	5	6	7	12
13	14	15	20	21
22	23	28	29	30

Card 4

8	9	10	11	12
13	14	15	24	25
26	27	28	29	30

Card 5

16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

NUMBER 13 PICK ANY NUMBER

TEACHER'S INSTRUCTIONS

Here is an example of the most common magic trick using numbers. Your subject thinks of a number and is asked to perform several arithmetic computations after which you make a prediction. In this trick, you will know the answer before your subject selects a number.

SKILLS

- Basic arithmetic computation
  addition
  subtraction
  multiplication
  division by 2

MATERIALS

- Paper and pencil or a calculator

PERFORMING THE TRICK

Ask your subject to think of a number, any number. (Since computation is involved, your subject should have paper and pencil, or if a large number is chosen, your subject may wish to use a calculator.) Ask your subject to carry out computations 1–4 below.

1. **Double your chosen number.**
2. **Add 10 to this result.**
3. **Divide the last sum by 2.**
4. **From this result, subtract the original number.**
5. **Since you know the result of these computations before the trick begins, you can reveal your prediction in any way you wish.**

HOW THE TRICK WORKS

The prediction is always 5. Before performing this trick, you can seal your prediction in an envelope or have it prominently displayed in the room. The problem with this trick is that the answer is always 5, which your subjects will realize if you repeat the trick.

There is an easy way to change your prediction. The prediction is always half of the number that is added in step 2 of the trick. Thus, if you want the prediction to be 15, you must tell your subject to add 30 in step 2. Once you know how to alter the prediction, you can repeat the trick many times with the same subjects. You must add an even number in step 2 or your prediction will be a mixed number.

AN ILLUSTRATION

Suppose your subject chooses the number 38. The computations are shown for each step of the trick.

STEP	COMPUTATION
1	$2 \times 38 = 76$
2	$76 + 10 = 86$
3	$86 \div 2 = 43$
4	$43 - 38 = 5$, which is your prediction.

Now, suppose you decide to change your prediction to 95. When you repeat the trick, simply replace step 2 with "add 190 [which is 2 × 95] to the result." The computations for this variation of the trick are shown below for a subject's choice of 365.

STEP	COMPUTATION
1	$2 \times 365 = 730$
2	$730 + 190 = 920$
3	$920 \div 2 = 460$
4	$460 - 365 = 95$, which is your new prediction.

WHY THE TRICK ALWAYS WORKS

Suppose we think of a set of objects, where □ represents the number of objects selected by your subject. The trick alters the number of objects in the set as shown below.

OPERATION ON THE SET	NUMBER IN THE SET
The original number in the set	□
Step 1, double the number	□□
Step 2, add ten to the set	□□ + 10
Step 3, take half of the set	□ + 5
Step 4, remove the original number from the set	5

Thus, it does not matter what number is selected by the subject, the result is always 5. Also, note that whatever number you add to the set in step 2, the result of step 4 will be half of that number.

USING THE TRICK

If you perform this trick in front of a class, students can choose their own numbers and use calculators to carry out their computations. Each student's computation will result in the answer 5, as you predict.

As you repeat the trick, ask students what they think the result will be. Since everyone got a 5 previously, they should predict the same result. Now, you can alter the number added in step 2 to change your prediction.

Using the activity pages (pp. 79–80), students can discover why the original prediction of 5 works and how this prediction can be altered. The observations illustrate the use of the set model to explain why the magician's prediction always works.

An exercise on activity page 80 challenges students to make alterations in the trick and then to create their own PICK A NUMBER trick.

PICK ANY NUMBER and the MATHEMATICS CURRICULUM

As in THE MONEY TRANSFER trick (p. 64), this activity is based on translating verbal or written phrases either into expressions using mathematical operations or into physical models. Below are some examples from the elementary mathematics curriculum.

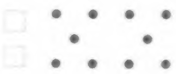
1. Use symbols to represent mathematical operations.

WRITTEN PHRASE	ARITHMETIC EXPRESSION	EXPRESSION USING THE SYMBOL □
choose any number	38	□
double your chosen number	2 × 38	□□
add ten to your result above	(2 × 38) + 10, or 86	□□ + 10

2. Using □ to represent the original number of elements in a set and the symbol • to represent individual elements in a set, the three phrases above can also be represented by the three sets below.

□

□□



ACTIVITY PAGES for PICK ANY NUMBER

After seeing this trick performed, you may have guessed what the prediction will be. See if you can discover why the magician's prediction is always correct and how the magician can change the prediction.

PICK ANY NUMBER TRICK

Ask your subject to think of a number, any number. (Since computation is involved, your subject should have paper and pencil, or if a large number is chosen, your subject may wish to use a calculator.) Ask your subject to carry out computations 1-4 below.

1. **Double your chosen number.**
2. **Add 10 to this result.**
3. **Divide the last sum by 2.**
4. **From this result subtract the original number.**
5. **Since you know the result of these computations before the trick begins, you can reveal your prediction in any way you wish.**

OBSERVATIONS

In the observations below, let the symbol ■ represent an unknown number of elements (the number selected by the subject in the trick) and the symbol ● represent single elements in a set.

1. Draw a circle to represent a set. In this set, place the unknown number of elements thought of by the subject.

2. Draw the set above after the number of elements has been doubled.

3. Draw the set from observation 2 after ten more elements have been added to the set.

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4. Draw the set above after you remove half of the elements.

5. Finally, draw the set from observation 4 after you remove the original number of elements that were in the set.

CONCLUSION

1. Predict how many elements are left in the set you drew above. Explain why the number selected by the subject does not change your prediction.

2. In observation 3 above, suppose twelve elements were added to the set instead of ten. Explain how this would change your prediction. If you wanted your prediction to be 35, what changes would you have to make in the directions to the subject? Now, explain how you could make any prediction that you want in this trick. How would adding an odd number in step 2 in the trick alter your prediction?

MAKING YOUR OWN TRICK

Use the set model from these activity pages to show a set with an unknown number of elements. Make several changes in the number of elements in your set, until you arrive at a predictable number of elements. Now, write a list of directions that explains the changes in your set. Use this set of directions to perform your own PICK ANY NUMBER trick.

NUMBER 14 NICKEL AND DIME

TEACHER'S INSTRUCTIONS

A subject conceals a nickel in one hand and a dime in the other. After a few computations, you announce which hand holds which coin.

SKILLS

- Multiplying by 2 and 3
- Multiplication rules:
  even times any number is even
  odd times odd is odd
- Addition rules:
  even plus odd is odd
  odd plus odd is even
  even plus even is even

MATERIALS

- A dime and a nickel

PERFORMING THE TRICK

Hand a nickel and a dime to your subject. For the remainder of the trick, turn your back to your subject. Give directions 1–7 below to your subject.

1. Hold one coin in your left hand and one coin in your right hand.
2. Multiply the value of the coin in your left hand by 2.
3. Multiply the value of the coin in your right hand by 3.
4. Sum the two products.
5. Can you divide this sum by 2?
6. If the answer is yes, divide the sum by 2.

   If the answer is no, add one and then divide by 2.

   Remember your answer.
7. Hold your two closed fists in front of you.
8. As you turn and face your subject, announce which hand holds the nickel and which the dime. Although you have a 50% chance of guessing correctly, you will always be right if you follow the rule below.

HOW THE TRICK WORKS

If the answer to the question in step 5 is "yes," then the dime is in the right hand; otherwise, the nickel is in the right hand.

AN ILLUSTRATION

Suppose your subject has put the nickel in his or her left hand and the dime in the right hand. Below are the computations done by your subject in each step of the trick.

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STEP	COMPUTATION
2	$5 \times 2 = 10$
3	$3 \times 10 = 30$
4	$10 + 30 = 40$
5	yes
6	$40 \div 2 = 20$

When you turn, touch your subject's left hand and say, "The nickel is in here, and the dime is in your right hand."

WHY THE TRICK WORKS

No matter what coin is in the left hand, when you multiply by 2, the product will be even. The product of an even number and any other number is always even.

When your subject multiplies the value of the coin in the right hand by 3, the product will be even for a dime (odd times even is even) and odd for a nickel (odd times odd is odd).

When these two products are added, the result is even if the dime is in the right hand (even plus even is even). The result will be odd if the nickel is in the right hand (even plus odd is odd). Thus, the answer to the question in step 5 identifies where the nickel and dime are located.

Notice that step 6 in the trick is totally unnecessary, but it diverts your subject's attention from the fact that you already know where the coins are.

USING THE TRICK

If you repeat this trick, the subject might feel that the computations are important to the magician's prediction even though the magician is never told the results of the computation. If you can make the question in step 5 of the trick seem like a computational rule, it will help you hide the "trick" from your subject. For example, you might say, "Divide by 2 if you can; otherwise, add one and then divide by 2. Tell me what computation you did."

Discovery of the mathematics behind the trick is explored on the activity pages (pp. 84–85). You can guide students through the activity pages or have them work individually or in small groups.

Many variations of this trick are possible and some of these variations are suggested in an exercise on activity page 85.

NICKEL AND DIME and the MATHEMATICS CURRICULUM

This trick makes use of the addition and multiplication rules for even and odd numbers. To understand the trick, students must be able to distinguish between even and odd numbers. In step 5 of the trick the subject is asked, "Can you divide the sum by 2?" The implication being, "Is your answer even or odd?"

When students can identify even and odd numbers, properties of even and odd can be explored. Below are a few illustration from the elementary mathematics curriculum.

1. Using whole numbers, give numerical examples to illustrate each statement below.
    a. The product of an even number and an odd number is even.
    b. The sum of two odd numbers is even.
2. Which of the following sentences is always true?
    a. The product of 3 and any other number is odd.
    b. The product of 2 and any other number is even.

ACTIVITY PAGES for NICKEL AND DIME

Although the subject never tells the magician the results of the computation, the magician knows where the coins are. See if you can discover how.

NICKEL AND DIME TRICK

Hand a nickel and a dime to your subject. For the remainder of the trick, turn your back to your subject. Give directions 1–7 below to your subject.

1. **Hold one coin in your left hand and one coin in your right hand.**
2. **Multiply the value of the coin in your left hand by 2.**
3. **Multiply the value of the coin in your right hand by 3.**
4. **Sum the two products.**
5. **Can you divide this sum by 2?**
6. **If the answer is yes, divide the sum by 2.**

   If the answer is no, add one and then divide by 2.

   Remember your answer.
7. **Hold your two closed fists in front of you.**
8. **As you turn and face your subject, announce which hand holds the nickel and which holds the dime.**

OBSERVATIONS

1. Suppose the subject holds the dime in his or her left hand and the nickel in the right hand. Use the table below to follow the computations in the trick. After each step, write whether the answer is an even or odd number.

STEP	COMPUTATION	EVEN OR ODD
2		
3		
4		
5	answer to question: ______	
6		

2. The only other possibility is that the subject will hold the nickel in the left hand and the dime in the right hand. Again fill in the table below.

STEP	COMPUTATION	EVEN OR ODD
2		
3		
4		
5	answer to question: ______	
6		

3. The sum in step 4 is either even or odd. When is it even? When is it odd?

CONCLUSION

1. Explain how the magician knows where each coin is hidden after the subject answers the question asked in step 5 of the trick.

2. Could the magician have used numbers other than 2 and 3 for multiplication?

3. Why do you suppose step 6 was included in the trick? Is this step necessary for making the prediction?

MAKING YOUR OWN TRICK

The magician knew where the coins were hidden based on the answer to the question in step 5 of the trick. Write your own trick where the prediction is based on "even and odd." You may wish to use several types of coins or design the trick without using coins. Maybe you could use a "think of a number" type of trick. Also, you might allow the subject to choose the two multipliers used in step 2 and step 3 of the trick.

NUMBER 15 CALENDAR ARITHMETIC

TEACHER'S INSTRUCTIONS

In this trick you will need a calendar—any year will do. Your subject picks any month of the year and outlines any 3 × 3 or 4 × 4 block of numbers for that month. Then your subject computes the sum of this block of numbers, but before your subject gets started, you will know the sum.

SKILLS

- Compute sums of two-digit numbers
- Multiply a two-digit number by 8 or by 9

MATERIALS

- A calendar (a reproducible 1995 calendar is provided on page 92)

PERFORMING THE TRICK

The 1995 calendar is used to illustrate how the trick is performed.

1. **Ask your subject to choose any month of the year and to draw a box around any 3 × 3 or 4 × 4 block of numbers in the chosen month. For example, suppose October 1995 is chosen and the 3 × 3 block of numbers shown below is outlined.**

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

2. **As soon as your subject begins outlining the numbers, you can write your prediction on a card and place it facedown on the table.**
3. **Now, ask your subject to sum the 9 numbers (or 16 numbers in a 4 × 4 block of numbers) in the box. When your subject finishes the sum, turn over the card and show that your prediction matches the sum. You do not have to sum the numbers in the box in order to make your prediction. If you follow the two simple rules below, your prediction can be made quickly, and with a little practice, it will always be correct.**

HOW THE TRICK WORKS

As soon as you see which block of numbers is being outlined, compute using the following rules:

- For a 3 × 3 block: multiply the middle entry by 9.

In the example above, your prediction is 9 × 20 = 180. You can do this computation with a pencil and paper. But, with a little practice, you can mentally compute this product both quickly and accurately. This makes the trick appear more mysterious.

- For a 4 × 4 block: add the two numbers in the diagonal corners of the block and then multiply this sum by 8. This computation is a little more difficult, but with practice you will be able to do it.

AN ILLUSTRATION

Consider the 4 × 4 block of numbers outlined below in the month of November 1995.

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Your prediction is computed as follows: $26 + 8 = 34$ and $8 \times 34 = 272$. You may write your prediction, 272, on a card and place it facedown on the table until the end of the trick. Also note that you could have added the numbers in the opposite corners, 5 and 29, to get the sum, 34. Meanwhile, your subject computes the sum of these 16 numbers—a calculator will speed up the trick. When your card is turned over, the prediction will match the calculator sum.

WHY THE TRICK ALWAYS WORKS

Suppose your subject selected a 3 × 3 block of numbers which starts with the 6th day of the month. This block would appear as follows:

6	7	8
13	14	15
20	21	22

The block of numbers could be rewritten as follows:

6	6 + 1	6 + 2
6 + 7	6 + 8	6 + 9
6 + 14	6 + 15	6 + 16

When you sum these nine numbers you have the following:

$$\begin{aligned} \text{SUM} &= 6 + (6 + 1) + (6 + 2) + (6 + 7) + (6 + 8) + (6 + 9) + (6 + 14) + (6 + 15) + (6 + 16) \\ &= (6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6) + (1 + 2 + 7 + 8 + 9 + 14 + 15 + 16) \\ &= (9 \times 6) + 72 \\ &= (9 \times 6) + (9 \times 8) \\ &= 9 \times (6 + 8) = 9 \times 14 \end{aligned}$$

Note that this sum is 9 times the number located in the center of the 3 × 3 block that was outlined.

The principle is the same no matter what the starting date is in the 3 × 3 block. Try the computation illustrated above using a different starting date.

Now, suppose your subject selected a 4 × 4 block starting with the 4th day of the month. This 4 × 4 block would look as follows:

4	5	6	7
11	12	13	14
18	19	20	21
25	26	27	28

This block can be rewritten as follows:

4	4 + 1	4 + 2	4 + 3
4 + 7	4 + 8	4 + 9	4 + 10
4 + 14	4 + 15	4 + 16	4 + 17
4 + 21	4 + 22	4 + 23	4 + 24

When you sum these sixteen numbers you get the following:

$$\text{SUM} = 4 + (4 + 1) + (4 + 2) + (4 + 3) + \ldots + (4 + 21) + (4 + 22) + (4 + 23) + (4 + 24)$$

$$= (4 + 4 + 4 + \ldots + 4 + 4) + (1 + 2 + 3 + 7 + 8 + 9 + 10 + 14 + 15 + 16 + 17 + 21 + 22 + 23 + 24)$$

$$= (4 \times 16) + 192$$

$$= (8 \times 8) + (8 \times 24)$$

$$= 8 \times (8 + 24)$$

$$= 8 \times 32$$

Note that this sum will be 8 times the sum of the two numbers in the diagonal corners of the 4 × 4 block of numbers outlined in the trick (either 28 + 4 or 25 + 7).

Again, this rule is the same no matter what the starting date is for the 4 × 4 block of numbers. Try this same computation using a different starting date.

USING THE TRICK

A calendar can be duplicated if you are working with an individual or small group of students. When performing the trick in the classroom, a transparency of a calendar may be used. A reproducible 1995 calendar appears on page 92.

If the calendar is projected from a transparency, your students can outline any 3 × 3 or 4 × 4 block of numbers in any month. As soon as you see the block of numbers being outlined, you can record your prediction in any way you see fit. Your students may use calculators to do their sums and then compare these sums with your predictions.

Students may discover the secret behind the calendar trick by completing the activity pages (pp. 90–91) or you can lead the class through this discovery process.

On activity page 91 is an exercise in which students try to reverse the calendar trick, that is, a subject picks a month and tells you the sum of the 9 or 16 numbers outlined and you draw a box around the block of numbers your subject has selected.

CALENDAR ARITHMETIC and the MATHEMATICS CURRICULUM

The magician's prediction is based upon number patterns found in a monthly calendar and mental multiplication by 8 or by 9. Below are a few exercises addressing these concepts and skills.

1. Each 3 × 3 or 4 × 4 block of numbers below is from a monthly calendar. Fill in the missing numbers.

a. 6 ___ ___
13 ___ ___
20 ___ ___

b. 15 16 17
___ ___ ___
___ ___ ___

c. ___ ___ 4 ___
___ ___ ___ ___
___ ___ ___ ___
___ ___ ___ ___

2. Use each of the rules below to complete the answer for the following three numbers: 14, 20, and 25.

Rule 1: Multiply the given number by 9.

Rule 2: Multiply the given number by 10 then from the result subtract the given number.

What do you notice about the answers using the two rules?

3. Use each of the rules below to complete the answer for the following three numbers: 19, 26, and 32.

Rule 1: Multiply the given number by 8.

Rule 2: Double the given number, then double the result, and then double the result again.

What do you notice about the answers using the two rules?

ACTIVITY PAGES for CALENDAR ARITHMETIC

As soon as a 3 × 3 or 4 × 4 block of numbers is outlined in this trick, the magician will know the sum of the numbers in the box. See if you can discover the magician's secret.

CALENDAR ARITHMETIC TRICK

1. Ask your subject to choose any month of the year and to draw a box around any 3 × 3 or 4 × 4 block of numbers in the chosen month. For example, suppose October 1995 is chosen and the 3 × 3 block of numbers shown below is outlined.

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

2. As soon as your subject begins outlining the numbers, you can write your prediction on a card and place it face down on the table.
3. Now, ask your subject to sum the 9 numbers (or 16 numbers in a 4 × 4 block of numbers) in the box. When your subject finishes the sum, turn over the card and show that your prediction matches the sum.

OBSERVATIONS

1. Write down several possible 3 × 3 blocks of numbers chosen from the calendar.

2. For each of these 3 × 3 blocks, compute the sum of the nine numbers.

3. Divide each of these sums by 9. Are the quotients whole numbers?

4. Compare each quotient with its 3 × 3 block of numbers. What do you notice about the quotients?

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CONCLUSION

1. Use your observations above to write a rule for finding the sum of the nine numbers in the 3 × 3 block.

2. For a 4 × 4 block of numbers, repeat the OBSERVATIONS and CONCLUSION sections above. However, in observation 3, divide each sum by 8 instead of 9.

MAKING YOUR OWN TRICK

Try making a trick that reverses CALENDAR ARITHMETIC. In this new trick, the subject identifies a month of the year and tells you the sum of a 3 × 3 or 4 × 4 block of numbers but does not show you the block that was chosen. In this trick, a calculator would be useful. You, as the magician, must determine which block of numbers the subject has chosen and outline these numbers. See if you can devise a set of rules for this trick.

To accomplish this, you must reverse the prediction rules for the CALENDAR ARITHMETIC trick. For example, in a 3 × 3 block, if you know the sum, then you will be able to determine the middle number in the block chosen by your subject. When you know the middle entry, you can outline the 9 numbers.

Similarly, for a 4 × 4 block of numbers, if you know the sum, then you will be able to predict the sum of the numbers in the diagonal corners. Since there are only a few possible 4 × 4 blocks of numbers for any month of the year, you should quickly be able to identify the correct 4 × 4 block and enclose it in a box.

1995 Calendar

JANUARY

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

FEBRUARY

			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28				

MARCH

			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

APRIL

						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

MAY

	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

JUNE

				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

JULY

						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

AUGUST

		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

SEPTEMBER

					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

OCTOBER

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

NOVEMBER

			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

DECEMBER

					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

100

NUMBER 16 THE DAY YOU WERE BORN

TEACHER'S INSTRUCTIONS

In this trick, you ask your subject to carry out a set of simple arithmetic computations. When the computations are completed, you will know the day, month, and year in which your subject was born.

SKILLS

- Add single-digit numbers
- Multiply two two-digit numbers
- Add two numbers where carrying is involved
- Subtract where renaming is involved

MATERIALS

- Pencil and paper or a calculator

Each instruction below is followed by the computations that would be done by a subject born on August 20, 1984.

PERFORMING THE TRICK

Instructions	**Sample Computations**
1. Write the number of the month you were born.	1. 8
2. Add the next largest number.	2. 8 + 9 = 17
3. Multiply this sum by 50.	3. 17 × 50 = 850
4. Add 73 to this product.	4. 850 + 73 = 923
5. Now, add the day of the month in which you were born.	5. 923 + 20 = 943
6. Write the digits 45 to the right of the digits in your last sum.	6. 94345
7. To your last number, add the last two digits of the year you were born.	7. 94345 + 84 = 94429
8. Tell me your last result.	8. 94429 is announced

If your subject is using a calculator, then the number in step 6 (94345 in the example) must be entered into the calculator. Keying the number alone (45) will erase the number which is on the display (943).

If your subject is using paper and pencil, the result in step 8 can be given to you verbally or written on the board. If a calculator is being used, your subject can hand you the calculator with the result on the display.

At the completion of step 8, you will know the day, the month, and the year in which your subject was born if you carry out the simple computation shown below.

HOW THE TRICK WORKS

From the answer given by your subject, 94429 above, subtract 12345. If your subject is using a calculator, simply subtract 12345 from the number on the display. If your subject used a pad and pencil, do your computation on paper or use mental arithmetic. The latter is more impressive as long as you do not make a mistake.

In the example above you get

$$\begin{array}{r} 94429 \\ \underline{-\ 12345} \\ 82084 \end{array}$$

Here is your prediction:

The last two digits, 84, identify the year of birth (1984).

The next two digits, 20, identify the day of the month (the 20th).

The remaining digit or digits, 8, identify the month (the 8th month, or August).

So, you can announce that your subject was born on the 20th of August in 1984.

ANOTHER ILLUSTRATION

Suppose your subject's birth date is December 9, 1982. Your subject will carry out the computations below.

INSTRUCTION NUMBER	COMPUTATION
1	12
2	12 + 13 = 25
3	25 × 50 = 1250
4	1250 + 73 = 1323
5	1323 + 9 = 1332
6	133245
7	133245 + 82 = 133327
8	133327 is announced

When the number 133327 is given to you, subtract 12345 as shown below.

$$\begin{array}{r} 133327 \\ \underline{-\ 12345} \\ 120982 \end{array}$$

The digits 82 tell you the year, 1982.

The digits 09 tell you the day, the 9th.

The digits 12 tell you the month, December.

At this point, you announce your prediction: Your subject was born on the ninth of December in 1982.

WHY THE TRICK ALWAYS WORKS

The seven computational steps of the trick are repeated for a person born on the 20th of August, 1984. However, this time the month (8), the day (20), and the year (84) are left in the computation until the last instruction is given. These three numbers are identified as follows: the month is circled; the day is boxed; and the year is underlined.

INSTRUCTION #	COMPUTATION
1	$\textcircled{8}$
2	$\textcircled{8} + (\textcircled{8} + 1) = (2 \times \textcircled{8}) + 1$
3	$50 \times ((2 \times \textcircled{8}) + 1)) = (50 \times 2 \times \textcircled{8}) + (50 \times 1) = (100 \times \textcircled{8}) + 50$
4	$(100 \times \textcircled{8}) + 50 + 73 = (100 \times \textcircled{8}) + 123$
5	$(100 \times \textcircled{8}) + 123 + \boxed{20}$
6	$100 \times ((100 \times \textcircled{8}) + 123 + \boxed{20}) + 45$
=	$(10000 \times \textcircled{8}) + 12300 + (100 \times \boxed{20}) + 45$
=	$(10000 \times \textcircled{8}) + (100 \times \boxed{20}) + 12345$
7	$(10000 \times \textcircled{8}) + (100 \times \boxed{20}) + \underline{84} + 12345$

From the number announced in step 7 above, you subtract 12345 and get the result: $(10000 \times \textcircled{8}) + (100 \times \boxed{20}) + \underline{84}$. Note that the last two digits are 84 (the year), the next two will be 20 (the day), and the first digit is 8 (the month).

USING THE TRICK

This trick can be done with an individual subject, a group of students, or an entire class. After students have completed the last computation and you know their final results, you can quickly announce the birth date for each subject.

After completing the activity pages (pp. 97–98), students will know the secret for predicting a person's birth date. Now, if students memorize the list of computational steps used in the trick, they can perform the trick themselves.

On page 98 of the activity pages is an exercise which allows students to develop another prediction trick based on similar mathematical skills.

THE DAY YOU WERE BORN and the MATHEMATICS CURRICULUM

As the month, day, and year are entered into this trick, the computations are designed to separate these numbers so the magician can make a correct prediction. The concepts of place value and inverse operations are used in the process. Below are some illustrations of these concepts from the elementary mathematics curriculum.

1. Circle the number below represented by the following computation:
   $(7 \times 10^3) + (4 \times 10^2) + (9 \times 10^1) + 0$.

   a. 749

  b. 4,790

  c. 7,490

2. Identify the digits in the thousands place and hundreds place for each of the following numbers:

   a. 3837

   b. 1042

   c. 958

3. Suppose you add 5 to a given number. What operation on the result will bring you back to the original number?

4. Starting with a given number, add 5 to the number and then multiply the result by 2. What two operations on the answer will bring you back to the original number?

ACTIVITY PAGES for THE DAY YOU WERE BORN

See if you can discover the prediction secret for this trick.

THE DAY YOU WERE BORN TRICK

1. Write the number of the month you were born.
2. Add the next largest number.
3. Multiply this sum by 50.
4. Add 73 to this product.
5. Now, add the day of the month in which you were born.
6. Write the digits 45 to the right of the digits in your last sum.
7. To your last number, add the last two digits of the year you were born.
8. Tell me your last result.

OBSERVATIONS

Write down the four to six numbers which represent the month, the day, and the last two digits of the year in which you were born.

1. Follow the seven computational steps listed in the trick above using the numbers for your birth date. Your result in step 7 above is the number you are asked to announce to the magician. From this number, the magician determines your birth date.

2. Under your answer in step 7 above, write the following digits:
   - Under the last two digits, write the last two digits of the year you were born.
   - Under the next two digits, write the digits of the day you were born—if this is a one-digit number, put a 0 in front of it.
   - Under the first digits, write the number of the month you were born.

   Now, subtract these two numbers. What is the result?

3. If you knew the computational result of the last step in the trick above and the result you just found in observation 2 above, how could you predict the subject's birth date?

CONCLUSION

1. Write a computational rule that could be used to predict a subject's birth date from the computational result of the last step in the trick.

2. Pick another birth date and test your rule stated in conclusion 1 above.

MAKING YOUR OWN TRICK

Here are the computational steps to another prediction trick. Following the directions below, try this trick on yourself. See if you can determine what you can predict from the result of the last step in the trick.

COMPUTATIONS

1. Think of any number and write it down.
2. Double this number.
3. Add five to the result.
4. Multiply your answer by 50.
5. Add your age.
6. From this sum subtract 365.
7. Finally, add 115.

How does the result in step 7 above allow you to tell your subject's age?

What else can you predict from the result in step 7?

Do you think this trick will always work?

See if you can discover why this trick will always work.

NUMBER 17 PREDICTING A PAIR

TEACHER'S INSTRUCTIONS

Here is an extension of the PICK ANY NUMBER trick (p. 76). Your subject picks two numbers (determined by rolling a pair of dice) and performs certain computations. When the computations are complete, you predict the two numbers rolled on the dice.

SKILLS

- Addition and subtraction
- Multiplication by 2 and by 5
- Distributive property
- Place value

MATERIALS

- A pair of dice (preferably two different colors)
- Paper and pencil

PERFORMING THE TRICK

Hand your subject a pair of dice. It will be helpful if you use dice of two different colors, but it is not necessary. (If you do not have dice of two different colors, you can refer to the two dice as the first die and the second die.) Let us assume one die is red and the other green.

Turn your back while you ask your subject to roll the dice then carry out computations 1–4 below.

1. Double the number showing on the red die.
2. Add 5 to this result.
3. Multiply this sum by 5.
4. To this product, add the number showing on the green die.
5. Ask the subject to tell you the answer. From your subject's answer, you can predict the numbers showing on the red die and the green die.

HOW THE TRICK WORKS

When your subject's answer is made known to you, mentally subtract 25. The first digit (tens place) of the answer is the number on the red die and the second digit (units place) is the number on the green die.

AN ILLUSTRATION

Suppose your subject rolls a 6 on the red die and a 3 on the green die. Your subject's computations are shown below.

STEP	COMPUTATION
1	$6 \times 2 = 12$
2	$12 + 5 = 17$
3	$5 \times 17 = 85$

4	$85 + 3 = 88$

When your subject tells you the result, 88, you subtract 25, which gives you the answer 63. Thus, you announce that there is a 6 on the red die and a 3 on the green die.

WHY THE TRICK ALWAYS WORKS

This trick can be explained in two ways. One way is to list all the possible rolls for a pair of dice (36 in all) and check the prediction for each of these possible rolls. Another way is to explain the prediction using place value and the distributive property of arithmetic. This method is illustrated below and is used on activity pages 102–3.

Suppose after the roll that the red die shows a 6 and the green die a 3. The computations are written below with the 6 and the 3 left in the computation until the end of the trick.

STEP	COMPUTATION
1	6×2
2	$(6 \times 2) + 5$
3	$5 \times ((6 \times 2) + 5) = (6 \times 2 \times 5) + (5 \times 5)$ by the distributive property $= (6 \times 10) + 25$
4	$(6 \times 10) + 25 + 3$

The answer in step 4 above is announced by your subject. When you subtract 25, the answer is $(6 \times 10) + 3$. The 6 (red die) is in the tens place, and the 3 (green die) is in the units place of your answer, thus allowing you to make your prediction.

USING THE TRICK

Even when you repeat this trick, it is difficult for your subject to determine how you can make your prediction. This is because the numbers rolled are not apparent in the subject's result until you mentally subtract 25 from this number. Students can discover how this prediction is made by completing the activity pages (pp. 102–3).

Because of the way in which this trick is presented, the two numbers used must be single-digit numbers. The use of dice in the trick ensures that the numbers are single-digit. However, you need not use dice. You can have your subject select a domino from a set and use the dots to determine the two numbers. Likewise, you can simply ask your subject to choose any two single-digit numbers.

On activity page 103 are suggestions for creating a trick that predicts two or more numbers where the numbers in question may not be single-digit numbers.

PREDICTING A PAIR and the MATHEMATICS CURRICULUM

The analysis of the trick on the activity pages (pp. 102–3) emphasizes moving the "red" number to the tens place and the "green" number to the units place. This configuration is accomplished using the distributive property of arithmetic and requires an understanding of place value. Some illustrations from the mathematics curriculum are presented below.

1. Write the expressions below in standard notation.
   a. 6 × 10 + 3
   b. 14 × 100 + 11
   c. 11 × 100 + 5
2. Identify the tens digit in 63.
3. Identify the units digit in 45.
4. Using the distributive property, fill in the blanks in the following sentences:
   a. 5 × (3 + 2) = (5 × ___) + (5 × ___)
   b. 5 × ((6 × 2) + 5) = (___ × 6) + (___)

ACTIVITY PAGES for PREDICTING A PAIR

See if you can discover how the magician predicts what numbers appear on each of the dice.

PREDICTING A PAIR TRICK

Turn your back while you ask your subject to roll the dice then carry out computations 1–4 below.

1. Double the number showing on the red die.
2. Add 5 to this result.
3. Multiply this sum by 5.
4. To this product, add the number showing on the green die.
5. Ask the subject to tell you the answer. From your subject's answer, you can predict the numbers showing on the red die and the green die.

OBSERVATIONS

Roll a pair of red and green dice and write the resulting numbers in the boxes shown to the right. For each observation below, fill in the boxes with the correct whole numbers.

☐ ☐
red green

1. Step 1 in the trick multiplies the "red" die by 2.

   ☐ × 2
   red

2. Step 2 adds 5 to the result.

   (☐ × 2) + 5
   red

3. Step 3 multiples the result by 5.

   5 × ((☐ × 2) + 5)
   red

4. Rewrite observation 3 using the distributive property.

$$\_\_\_\_ \times (\underset{\text{red}}{\boxed{}} \times \_\_\_\_) + (\_\_\_\_ \times 5)$$

5. Simplify your answer in observation 4.

$$\underset{\text{red}}{\boxed{}} \times \_\_\_\_ + \_\_\_\_$$

6. Step 4 adds the "green" number.

$$\underset{\text{red}}{\boxed{}} \times \_\_\_\_ + \_\_\_\_ + \underset{\text{green}}{\boxed{}}$$

7. Subtract 25 from the result of observation 6 above and draw an appropriate diagram for the result.

CONCLUSION

1. Use your answer from observation 7 above to explain how the magician can predict the numbers showing on the red and green dice.

MAKING YOUR OWN TRICK

In the EXPERIMENTAL DICE trick, polyhedra dice were discussed (see page 62). Suppose ordinary dice were replaced by icosahedra dice (numbered 1 to 20) or dodecahedra dice (numbered 1 to 12) in the PREDICTING A PAIR trick. When these dice are rolled, the numbers showing on the red and green dice may not be single-digit numbers.

Alter the four computational steps in the trick so that you will be able to predict the numbers rolled on these new dice. You may also consider extending the trick by having your subject roll three dice.

NUMBER 18 FOUR AND FIVE

TEACHER'S INSTRUCTIONS

In this trick, the subject picks a two-digit number, then picks a country, and then picks an animal, all unknown to the magician. Yet, the magician seems to read the subject's mind.

SKILLS

- Place value: units and tens
- Subtraction of two-digit numbers
- Multiples of nine

MATERIALS

- Pencil and paper

PERFORMING THE TRICK

Give directions 1–8 below to your subject. Paper and pencil should be available for some simple computation.

1. Think of a two-digit number which has two different digits.
2. Form a second two-digit number by switching the units digit and the tens digit.
3. Subtract the smaller two-digit number from the larger.
4. If there is more than one digit in your answer, sum the digits. Otherwise, leave your answer as it is.
5. Subtract 5 from the last result.
6. Convert your answer into a letter of the alphabet by letting the number 1 be A, 2 be B, 3 be C, and so forth.
7. Think of a country whose name begins with that letter.
8. Take the next letter in the alphabet and think of an animal whose name begins with that letter.
9. At this point, ask your subject, "Were you thinking of an elephant in Denmark?" If this is the first time your subject has seen this trick, your prediction is usually correct.

HOW THE TRICK WORKS

No matter what two-digit number is selected in step 1, your subject arrives at the answer 4 in step 5 of the trick. Therefore, the two letters determined in steps 6 and 8 are "D" and "E." The most likely choices for a country and animal are "Denmark" and "elephant."

AN ILLUSTRATION

Suppose your subject chooses 37 for the two-digit number. The table below follows the subject through the eight steps in the trick.

STEP NUMBER	SUBJECT'S COMPUTATIONS AND RESPONSES
1	chooses 37
2	forms 73
3	73 – 37 = 36
4	3 + 6 = 9
5	9 – 5 = 4
6	converts 4 to D
7	You hope your subject selects Denmark.
8	next letter is E
9	You hope your subject selects elephant.

If your subject selects the two words you expect, the prediction seems quite amazing. You appear to read your subject's mind.

WHY THE TRICK WORKS

When any two-digit number is reversed and the difference between the numbers is computed, the result is either 9 or a two-digit multiple of 9. Two cases are illustrated here: (1) for a choice of 12, 21 – 12 = 9 and (2) for a choice of 79, 97 – 79 = 18. There are thirty-six possible subtractions (without using the digit 0) in the trick, each of which leads to a multiple of 9.

When the computed difference is a two-digit multiple of 9, the sum of these digits (step 4 of the trick) will be nine. This fact becomes obvious when you list the multiples of nine: 18, 27, 36, 45, 54, 63, 72, and 81. This means that your subject's answer in step 5 of the trick is always 4 since 9 – 5 = 4. Therefore your subject's letters in steps 6 and 8 are always "D" and "E." The country and animal that most people choose, especially if you do not give them a long time to think, are "Denmark" and "elephant." Of course, someone might choose the "Dominican Republic" and "ewe." But, the first time you try this trick with a subject, you have a good chance of making the correct prediction.

USING THE TRICK

The key to predicting "elephant in Denmark" is based on forcing your subject to arrive at the answer 9 in step 4 of the trick. Again we are using one of those amazing properties of nine.

When you perform this trick, you take a risk, that is, it is possible that your prediction will not be correct. But, it is worth the risk since you will usually be right, and the overall effect of the trick is great. The secret behind this trick is explored on the activity pages (pp. 107–8).

This is the type of trick you should not repeat with the same subjects, and it is possible that some students have seen it performed before. You can fool even these students by changing the end of the trick. This possibility is explored as an exercise on activity page 108.

FOUR AND FIVE and the MATHEMATICS CURRICULUM ·

Mathematical explorations of some of the properties of nine were illustrated in the MAGIC NINE trick (see page 35). Here are a few other ways you can force the answer 9 or a multiple of 9.

1. Select any three-digit number in which none of the digits are the same. Form a different three-digit number by rearranging the original three digits. Compute the difference of these two numbers. Is the difference divisible by 9?
2. Write down any number with two or more digits. From this number subtract the sum of the digits in the original number. Is the difference divisible by 9?

ACTIVITY PAGES for FOUR AND FIVE

If the magician guessed your country and animal, you must have been amazed. See if you can discover how this was done.

FOUR AND FIVE TRICK

Give directions 1–8 below to your subject. Paper and pencil should be available for some simple computation.

1. Think of a two-digit number which has two different digits.
2. Form a second two-digit number by switching the units digit and the tens digit.
3. Subtract the smaller two-digit number from the larger.
4. If there is more than one digit in your answer, sum the digits. Otherwise, leave your answer as it is.
5. Subtract 5 from the last result.
6. Convert your answer into a letter of the alphabet by letting the number 1 be A, 2 be B, 3 be C, and so forth.
7. Think of a country whose name begins with that letter.
8. Take the next letter in the alphabet and think of an animal whose name begins with that letter.
9. At this point, ask your subject, "Were you thinking of an elephant in Denmark?" If this is the first time your subject has seen this trick, your prediction is usually correct.

OBSERVATIONS

1. Write down six two-digit numbers, each with two different digits. For each number, reverse the digits and subtract the smaller two-digit number from the larger. What do you notice about the six results?

2. There are 36 possible subtractions you could have carried out in observation 1 (if the digit 0 was not used). List all of these possible subtractions. With a little help from your friends, carry out each of these subtractions. What do you notice about all of the results?

3. Write down all the two-digit multiples of 9. Sum the two digits in each of these multiples. What do you find?

4. Can the magician predict what the subject's answer is in step 5 of the trick? How?

CONCLUSION

1. Explain why the magician knows the two letters used by the subject and why the magician feels safe in predicting "elephant in Denmark." Are there other possible solutions?

MAKING YOUR OWN TRICK

In the FOUR AND FIVE trick, the subject is forced (because of the computation) to choose the letters "D" and "E." Design your own trick where you force your subject to choose some letter (or letters). Have your subject choose words that begin with that letter (or letters), but make the answers predictable to you. Try not using countries or animals in your trick.

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